

A cellular automata intraurban model with prices and income-differentiated actors

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Abstract. This paper presents an intraurban cellular automata model that is an extension to White and Engelen's pioneering model. The paper's main contribution is to distinguish between agglomerative effects, determined by the attraction of the neighbourhood, and disagglomerative effects, driven by land prices, or land affordability. In order to do that, social heterogeneity is introduced in the model at the intraurban level. As a result, we can simulate both the evolution of land use and land prices. An application of the model and a sensitivity analysis indicate that neighborhood influence is the main driving force of cities' spatial configurations. Prices, however, exert an important countereffect. Actually, the higher the influence of land prices, the faster land succession is observed. Finally, an important conclusion of the model is that intraurban models should not fail to differentiate actors by income level.

Keywords: cellular automata, intraurban analysis, land prices, income-differentiated urban actors, urban modelling

1 Introduction and literature

Cellular automata (CA) represent one of the main ways of applying the self-organizing systems approach to urban models of land use and transport. Specifically, for urban matters, White and Engelen (1993), White et al (1997), Batty (1998), Torrens (2002), Pines and Thisse (2001), Capello (2002), Longley and Batty (2003), and Glaeser et al (2005) acknowledge CA as a promising instrument to deal with local interactions and social neighbourhoods, spatial irreversibilities, cumulative processes, and a variety of behaviours and urban space uses. Specifically, Brown et al (2005) and Batty (2005a) highlight the usage of CA models in studying processes, as opposed to forecasting them. Essentially, CA models have been used to achieve an understanding of a human-driven process that is basically dynamic and spatial.

Heterogeneity of agents in a modelling framework is urged by Fernandez et al (2005), who believe that models should reflect both the "heterogeneous nature of preferences" of agents and "diversity in the population". Heterogeneity of agents is also used in a CA

nonurban context by Dietzel and Clarke (2006) and van Delden and Hagen-Zaker (2009). Lau and Kam (2005) acknowledge the need for the inclusion of negative effects. However, they implement this with what they call a “retarding effect”, which is operationalized as inertial (gravity) forces.

Concerning the application of CA to urban, regional, and transport analysis, one of the seminal articles is that by White and Engelen (1993), which served as a conceptual basis for a series of subsequent papers (Barredo and Demicheli, 2003; Barredo et al, 2003; de Nijs et al, 2004; Engelen et al, 1995; Lau and Kam, 2005; Liu and Phinn, 2003; White et al, 1997) and the Metronamica modelling framework (RIKS, 2005; 2007a). In their proposed model, White and Engelen (1993) delineate the evolution of urban land over time by assuming a high spatial resolution in which the main interaction happens among actors (represented by land use) in a given neighbourhood. There are different land-use types (vacant, residential, industrial, and commercial), and the cells are converted from one use to another at each time step according to a set of transition rules. With a fairly simple model, White and Engelen (1993) simulate the land-use dynamics that structure urban environments. Some models (Engelen et al, 2004; RIKS, 2005; White et al, 1997; Wickramasuriya et al, 2009) are explicit extensions of the seminal work by White and Engelen in which the transition potential is determined not only by the neighbourhood effect, but also by a wider range of factors including dynamic suitability, accessibility, and zoning.

Other authors, such as Page (1999), Bell et al (2000), and Behrens (2005), follow similar methodologies and develop models in which there is a greater behavioural diversity. Portugali (2000) focuses on modelling segregation and migration processes. There have been a number of other models in the literature that focus on different aspects and emphasis. Some of them are mentioned here to help contextualize the model proposed in this paper.

Ward et al (2000) work on an urban growth model in which natural constraints and institutional controls are used to study possible patterns of future growth. Their model uses seeds and projection of population to simulate developed or undeveloped areas. Their scenarios can be used by the modeller to test public policy making, much like the White and Engelen (1993) model does. Another growth model is that of Torrens (2006), whose model describes the geography of sprawl based on the trajectory and movement of individual agents.

Benguigui et al (2001), in turn, develop a model in which leapfrogging growth is important. As a result, the model can reproduce undeveloped areas that are between the growing cities. This model has some similarity with ours in which urban emptiness is a central point. However, whereas Benguigui et al focus on a system of cities, our model characterizes leapfrogging within an intraurban environment.

Arai and Akiyama (2004) aim at identifying the explanatory factors driving the conversion from nonurban to urban land-use change, whereas Liu and Phinn (2003) present in a very didactic manner how CA models operate. Liu and Phinn’s contribution follows a linguistic approach in which the model translates qualitative analysis into parameters of the simulation. Further, Liu and Phinn differentiate each cell according to grades of membership into different categories (urban \times nonurban). Li and Yeh (2000) also consider the notion of ‘grey cells’—with grades of membership—but within a context that provides tools for a central planner to manage city growth into degrees of desired compactness.

Batty et al (1999) present a class of models in which typical urban activities spawn other activities within a life-cycle perspective. Their model is similar to ours in the sense that there is a predetermined not time-dependent transition matrix through which inertia is enforced by a strong diagonal. In doing so, their model is able to depict invasion and succession of land use, as proposed by Burgess (1925).

Wu and Webster (1998) specify that the model proposed—and others within the literature—are not adequate for prediction, but for visual comparison. Their model contains only one

land use and its emphasis is on the multicriteria used for the decision of the parameters of the model. Semboloni's (1997) model focuses on the market mechanism which governs land-use competition. Like our model, it calculates the rent value using the average of the price of the neighbours plus a segregation/preference parameter. Along with Wu and Webster (1998), Semboloni does not suggest a perfect forecast; instead emphasizing the usage of the model for policy-making purposes while clarifying the difficulty of calibrating such models towards a unique absolute optimum which does not exist.

Some other models refer to intraurban dynamics. Benenson et al (2002) use a very small neighbourhood constraint determined by the nearby houses with an emphasis on the socioeconomic characteristics of each household. A similar approach, with a three-dimensional configuration of many markets, is taken in the model by Semboloni et al (2004). O'Sullivan (2009) reinforces the need to focus on the neighbourhood, the interactions among neighbourhoods, and the agents' perception of them, much like the conceptualization made by Lynch (1960). This line of detailed urban scale also has the contribution of the work of Torrens (2007) and his emphasis on residential mobility and ontology of the model.

The important notion of distance to various places of interest observed in urban economics is incorporated in the model of Torrens and Nara (2007), along with the notions of gentrification. Also borrowed from the regional economics tradition is the model of Zhao and Murayama (2011) which uses a gravity model based on Tobler's first law of geography.

None of these approaches, however, takes the step of tying the intraurban scale back to aggregate dynamics. They also do not differentiate between intraurban land uses which interact differently among themselves.

In sum, we believe that current literature on urban CA modelling does not include: (a) intraurban socioeconomic heterogeneity, and (b) negative feedback of real-estate prices. These factors are not explicit in White and Engelen's model (1993), in which only locational quality, as defined by neighbouring land uses, determines land-use conversions. This paper also conceives the city as a multidimensional object that is the result of the individual actions of various agents (or actors, the preferred term in the urban planning literature⁽¹⁾) in space. Specifically, both Dietzel and Clarke (2006) and Fernandez et al (2005) demonstrate the importance of using distinct agent classes when modelling within an urban environment. Hence, we present a CA intraurban model that aims at differentiating agglomerative effects from disagglomerative ones within an income-differentiated, and thus heterogeneous, actors' framework.

Finally, it is also important to highlight that the sensitivity analysis may be seen as innovative in the sense that it helps understand the core of the model usage: that is, the support in clarifying intraurban heterogeneity dynamics. Further, the model uses spatial metrics analysis, as suggested by Herold et al (2005), and others (Hagen-Zanker, 2006; McGarigal et al, 2002). We could also try to frame the proposed model within the framework of assessment proposed by Pontius et al (2008). Our model would fall into the 'exogenous quantity', 'cellular automata' concepts, without usage of information after the calibration period. However, there are some large differences between our proposal and the studies depicted by Pontius et al. Firstly, the timespan of our model—103 years—is far beyond the maximum studied by them of only 43 years. Secondly, our starting period is nearly all empty space. Thirdly, the model does not aim at predicting exact land use, but rather the relationship developed within different intraurban actors.

The paper is organized with this introduction, followed by section 2 where the proposed extended model is presented and detailed. The case study for the metropolitan area of Belo

⁽¹⁾ In this paper, 'actor' is the term used for land use or cell states. In this context, an actor is equivalent to a cell. Therefore, a high-income land use, for example, is said to be a high-income actor.

Horizonte in Brazil in section 3 includes the description of the workspace and the actual dynamics and configurations of the model, the calibration, and the choice of parameters. Section 4 presents the results, the validation, and the insights provided by the sensitivity analysis. Some concluding remarks close the paper in section 5.

2 The proposed extended model

The extended model strengthens the theoretical foundations of the original White and Engelen model because it incorporates concepts of urban economic theory, such as centripetal and centrifugal forces (Colby, 1933; Krugman, 1996), that influence location behaviour. In other words, agglomerative and disagglomerative forces can be simply described as the attractiveness of a location to other points of interest (the centripetal force) weighted by diseconomies resulting from increasing prices due to competition for space (the centrifugal force).

Including both effects in the basic CA model requires that the conversion of land from one use to another be determined by (a) the presence of other urban land uses in the neighbourhood, weighted by distance (the neighbourhood effect), and (b) the price (land value) of the location. At the same time, both the neighbourhood effect and prices will change due to changes in the urban structure from these conversions. This suggests an iterative process, in which land-use conversions, on the one hand, and neighbourhood effects and prices, on the other hand, are mutually influential.

In a dynamic sense, actor locations at time t determine the land-use configuration and the associated land prices. Actors may generate a positive agglomeration effect for a specific site. However, the agglomeration advantage leads to more competition for the site, which leads to higher land prices (as well as congestion, pollution, etc), thus generating disagglomerative effects. Both effects are taken into consideration at the next time step in which there is allocation of new-entry actors. Competition among actors for locations following the transition rules (detailed below) leads to a new configuration of actors at $t+1$ which, in turn, establishes new land prices (figure 1).

Thus, prices reflect the attractiveness of locations (expressed in neighbourhood effects and accessibility) as the outcome of competition between actors for these locations. This implies that prices are related to the locational characteristics of the neighbourhood and its accessibility. In the proposed model extension, price will therefore be implemented as such.

The model proposed falls into the “exogenous quantity” classification suggested by Pontius et al (2008) in which the modeller provides the quantity of each category at each time step.

2.1 Land-use allocation

The model used in this study assumes, similar to White et al (1997), that what is being simulated is the spatial allocation of urban actors rather than their growth. Thus, the number of actors (land use) of each type is exogenously provided for each time step of the simulation as the outcome of municipal growth. The allocation of user types to cells and the transition of cells into a new state (land-use type) occur as follows:

- (1) In every time period of the simulation, the transition potential of all cells for all actor types is calculated. Thus, at every step there are as many transition potential maps as there are actor types being modelled.
- (2) The cell with the highest potential value among all transition potential values, for all actors, is allocated to the corresponding actor.
- (3) Once one transition occurs, the demand of that actor for that time period is reduced accordingly.

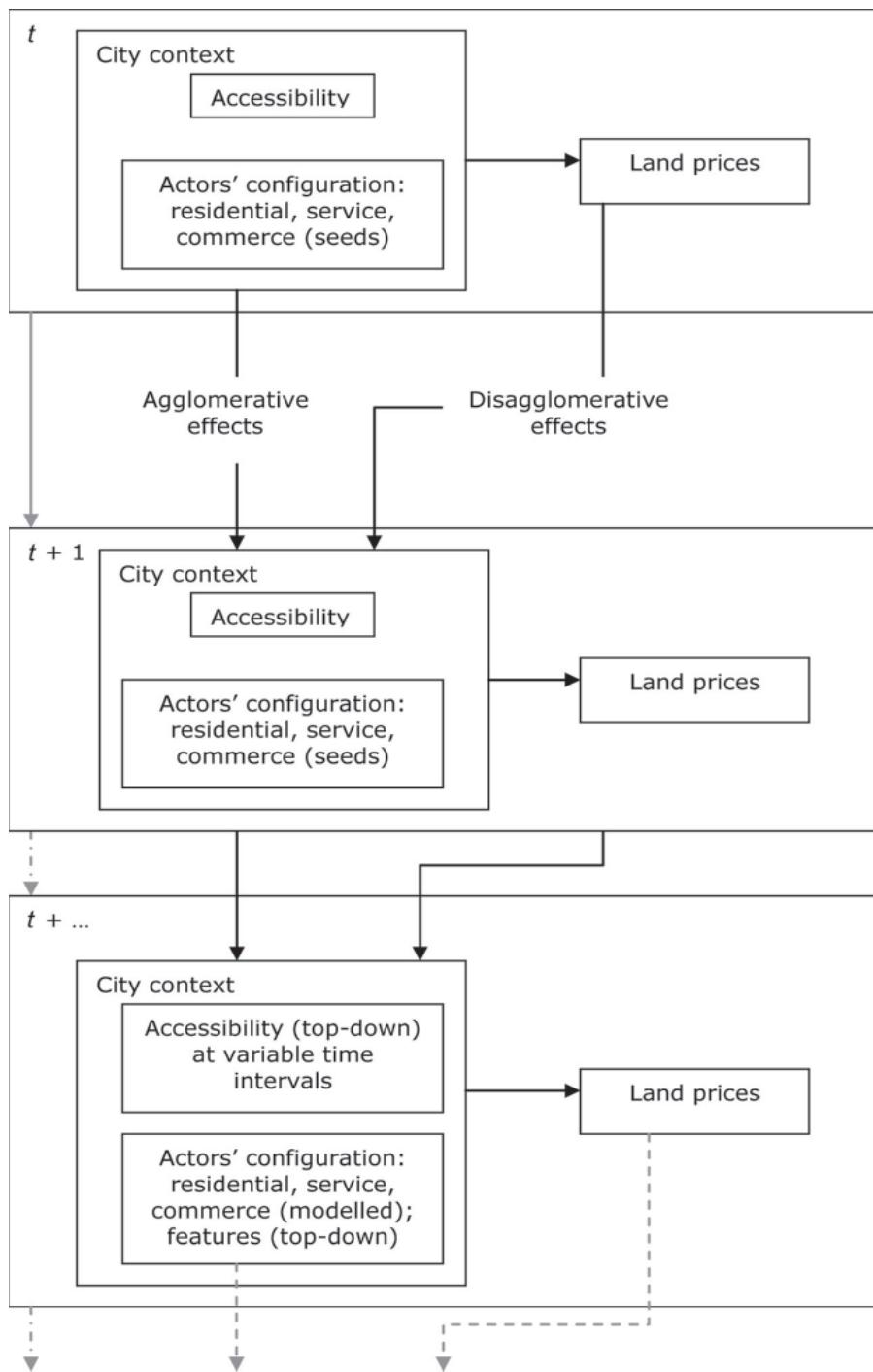


Figure 1. Illustration of processes.

- (4) Then, the second highest potential cell among all cells is allocated to the corresponding actor.
- (5) This process is repeated successively until all demands for a certain time period are met.

This allocation procedure implies that actor types compete for locations in a way that is governed by the price mechanism and spending power of the actors. If, ignoring land prices, a location is equally attractive to two actor types, differences in price sensitivity (as expressed by parameter τ_s , detailed below) will cause a higher preference for the location of the actor that is least price sensitive. Consequently, in such cases the actor with the largest spending power will occupy the location.

2.2 Total transition potential calculus

The total potential is expressed as the result of agglomerative effects minus disagglomerative effects [equation (1)]. Agglomerative effects include the influences of the neighbourhood and the total accessibility, whereas disagglomerative effects consist of the influence of location on price, weighted by spending power. In both effects a stochastic disturbance term is incorporated, representing the unobserved and unpredictable effects that exist among others as a result of heterogeneity in the likes and dislikes of each actor group.

Formally, we have

$${}^t T_{s,c} \equiv [(1 + e) {}^t N_{s,c} {}^t A_{s,c}^{\text{total}}] - [\beta e + (1 - \beta) {}^t P_c] \tau_s, \quad (1)$$

in which, ${}^t T_{s,c}$ is the transition potential value of each actor s , for each cell c at each time step t ; ${}^t N_{s,c}$ is the neighbourhood effect (detailed below), and ${}^t A_{s,c}$ (0,1) is the total accessibility value, both at each cell c , at a certain time t and for each actor (s); ${}^t P_c^{\text{total}}$ (0,1) is the price of cell c at time t , influenced by the neighbourhood; β [$\beta \in (0, 1)$] is a parameter that expresses the impact of factors not included in the model on real-estate prices, such as the estate's attributes and characteristics, its surface area and its quality; e are random values drawn from a uniform (0,1) distribution. τ_s is detailed below.

The idea of price parameters—central to the model—is that a fundamental factor to influence the locational choices of actors is their spending power. The parameters should then reflect the intensity of this effect on the model. The influence of prices on locational choices is divided into two moments.

First, the moment of allocation of entrant actors in the model (moving in), in which it is easier for wealthier families to actually afford and move into their place of choice. Therefore, τ_s [$\tau_s \in (0, 1)$] is a parameter that represents the impact of price on the transition potential, which depends upon the spending power of each class of actors. It is implicit that wealthier actors will be less affected by the negative impact of high prices. Their panorama of possible choices is wider.

Second, the fact that a given actor occupies the land influences the price of the land differently. Therefore, parameter μ_s conveys the idea that high-income previous residents influence nearby areas to higher prices. Although the resulting land price is the same for all actors, different μ_s values imply different influences on price formation.

Thus, land price, P , is calculated endogenously as a function of previous neighbourhood effect, actor's occupation (land use), and accessibility, as follows:

$${}^t P_c = \sum_{c' \in D(c)} \mu_{s(c')} , \quad (2)$$

where ${}^t P_c$ is given by the summation for a given radius, $D(c)$, of the actual configuration of actors at the previous time step, weighted by the parameter μ which varies for each actor s .

Occasionally, there are changes in accessibility as a result of policy implementation and funding availability, which is therefore top-down. There are also actors which are implemented exogenously—called features—in specific years given historical information. For each time step t , the above model will calculate the occupation of each cell (that is, which actor is located in the cell) as well as the price of the cell.

2.3 The neighbourhood effect, N

The neighbourhood effect is essential in CA models because it provides the tools to implement local spatial relations. The neighbourhood effect represents the impact of the perceived surroundings on the attractiveness of a cell for a certain use as well as its resistance to change:

its inertia. The neighbourhood effect is given by ${}^tN_{s,c}$. More specifically:⁽²⁾

$${}^tN_{s,c} = \sum_{c' \in D(c)} n_{s(c), s'(c'), d(c, c')} , \quad (3)$$

where $D(c)$ is the neighbourhood of cell c ; $d(c, c')$ is the Euclidean distance between cell c and cell c' ; $n_{s(c), s'(c'), d(c, c')}$ is the influence parameter n that expresses the strength of the influence of a cell c of a given actor $s(c)$ on another actor's cell $s'(c')$ for each distance $d(c, c')$. As n is specified for each possible combination of s , $s(\cdot)$, and d , we have a matrix of distance-decay functions. The user enters these in the model via the influence table that describes, for each pair of functions, how the impact on the transition potential depends on distance. See the example in figure 2.

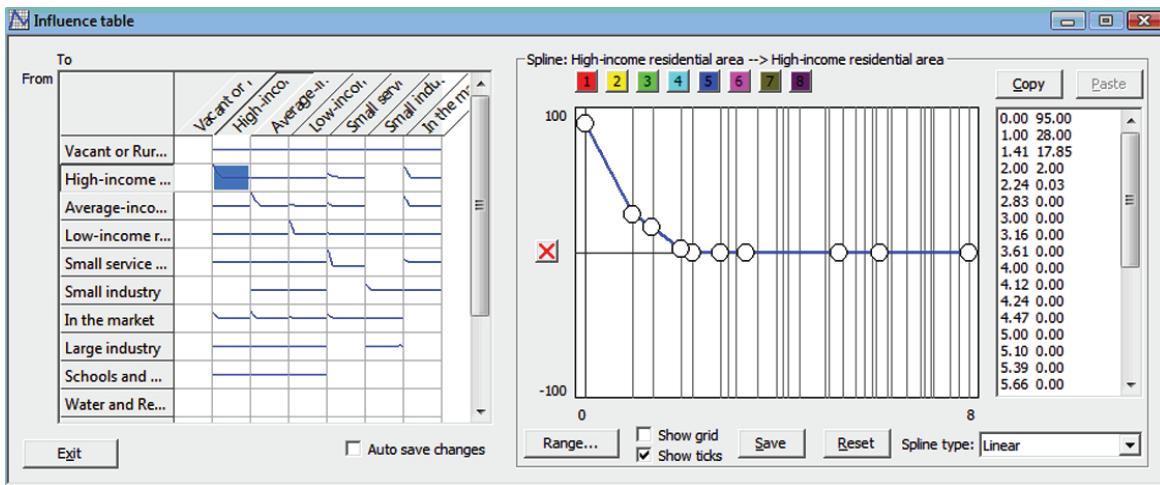


Figure 2. [In colour online.] Illustration of influence-table effects.

These parameters are determined iteratively and manually. This means that the model is run from time 1897 to 1991 (as shown in figure 4 below), the results are observed visually and the parameters in figure 2 are adjusted. The model is run again until the visual results and spatial metrics are defined as final.⁽³⁾ This approach differs from statistical 'brute force' methods in which the aim is to mimic land-use change (from rural or nonurban to urban) trends through machine learning-processes such as those incorporated in SLEUTH (Silva and Clarke, 2002). In the case of SLEUTH, the emphasis is on prediction for short periods of time. The model proposed in our case aims at extracting processual information of the interaction among different actors within an intraurban context.

In figure 2 the highlighted square on the left-hand side represents the influence of high-income residential areas on high-income residential areas. The graph and the specification on the right indicate that at distance 0, the impact is 95. This represents the impact of the cell on itself, which is, in fact, the inertia effect. The high value implies that a high-income residential area has a high probability of remaining a high-income area.

⁽²⁾ Adapted by the authors, from METRONAMICA model description (RIKS, 2007a).

⁽³⁾ This calibration process is also used by Pontius et al (2008), Liu and Phinn (2003), Semboloni (1997) and Herold et al (2005). Note that this calibration process is very informative in the sense of which and how individual parameters and relationships affect the model and supposedly the reality it represents.

2.4 Accessibility, A

Accessibility is a factor that brings the transport infrastructure and its evolution into the framework of the system. It accounts for proximity to the nodes and links of the infrastructure network and their impact on different types of actors. For every feature of accessibility and each of the modelled function land-use classes, the modeller provides two inputs: (a) a distance-decay function and (b) a relative importance factor for a certain type of network or node to each specific state ($W_{y,s}$). This is because different actors might value transport networks differently (such as bus systems or metro lines which are more important for low-income residential actors and arterial roads for private car users).

Formally, the equation for accessibility can be expressed as:

$${}^t A_{c,y,s} = \frac{a_{y,s}}{{}^t d_{c,y} + a_{y,s}}, \quad (5)$$

where ${}^t A_{c,y,s}$ is the accessibility of cell c in relation to a certain type of node or transport link y , for an specific actor s at time t ; $a_{y,s}$ is the accessibility distance-decay factor for actor s and type of node or transport link y , such as the importance of highways to small industry activity; and ${}^t d_{c,y}$ is the shortest distance, d , between a certain cell c and a transport node or link y at time t . The total accessibility value, which takes into consideration distance to all nodes and link types ($y \in Y$), is given by ${}^t A_{s,c}^{\text{total}}$, as

$${}^t A_{s,c}^{\text{total}} = \frac{1 - \left[\prod_{y \in Y} (1 - w_{y,s} {}^t A_{c,y,s}) \right]}{1 - \left[\prod_{y \in Y} (1 - w_{y,s}) \right]}. \quad (6)$$

In practice, this means that every cell has, for every actor, a total accessibility value that varies between 0 and 1.

3 Case study

3.1 Workspace

The workspace is defined as the municipality of Belo Horizonte and its neighbours (Betim, Contagem, Ibirité, Nova Lima, Ribeirão das Neves, Sabará, Santa Luzia, and Vespasiano), which form the Greater Belo Horizonte Area (figure 3).

The lattice or grid of the workspace contains 795 rows and 786 columns of which 288 592 are within the modelled area composed of the constituent municipalities. In the first year of the simulation, 1897, 33 individual cells, or 0.01% of the total were occupied by 'institutional buildings'. These represent initially present actors, such as the municipality's central buildings and urban infrastructure. In the literature these are usually described as seeds (Clarke et al, 1997). Around 3%, or 9892 cells, were already occupied as parks, green areas, and conservation units in 1897. These cells are called 'excluded areas' by Clarke et al (1997).

The spatial resolution is chosen such that cell size allows us to capture the differences and peculiarities of the intraurban space at the block level. Cell sizes in applications of urban models usually vary between 50 m and 150 m (Hagoort, 2006; Waddell et al, 2007). However, caution should be taken to not impede generalization and the global character of the simulation (RIKS, 2007b). In this case study, cells of 86 m by 86 m are used (roughly the size of a typical block in Belo Horizonte),⁽⁴⁾ which implies a surface area of 7400 m², or about 3/4 ha. The data used to initiate and validate the model are census data given at the level of census tracts, which are larger than the cells used.

⁽⁴⁾ 86 × 86 m was the minimum size of data availability.

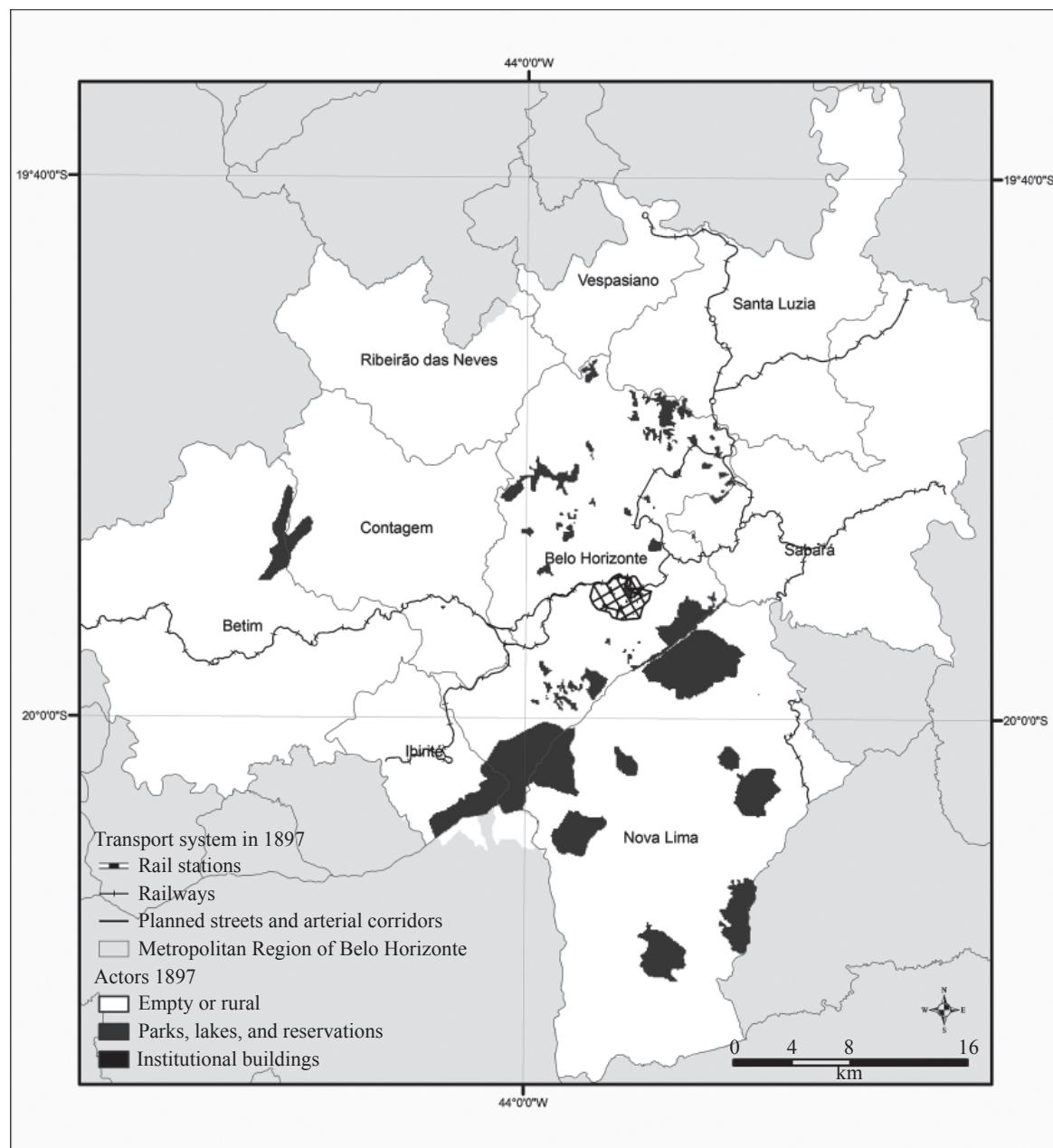


Figure 3. Workspace initial occupation (1897).

Additionally, census data do not contain information about nonresidential actors, which implies that information on the locations of service, commerce, industry, and 'in the market' (for sale) needs to be obtained from other sources. Although other sources (City Hall Tax database/PBH-SMFA, 2001) provided aggregated information (for example, the number of establishments by neighbourhood), they do not provide their exact location. Based on proportions derived from this database, the demand of these actors is designed so that 1% of the total occupied area is for small commerce and services, and 0.5% of the total occupied area is reserved for small industry. That is, the proportional quantity of cells allocated in each year of the simulation is predetermined according to the proportion observed in the database. However, spatial location is given by the mechanisms described in the model. Furthermore, the function in the market should account for the estates available in each period plus new allocations in each year such that, even after the newcomers are allocated, 1% still remains unoccupied.

In practice, what this means is that for the validation only the residential allocation resulting from the simulation is actually compared with real data. More specifically, the results of the simulation for residential actors are compared with data that includes only residential actors and exogenous features described in the next section. Given the essential role of the other nonresidential actors in the dynamics of the urban structure, however, they had to be part of the process even if it did not allow comparison of the data.

3.2 Software

The simulation is implemented using Metronamica.⁽⁵⁾ Metronamica is a spatial dynamic land-use modelling framework developed within the Research Institute for Knowledge Systems (RIKS), based on White and Engelen's (1993) paper. The system is defined as "a modeling and simulation package for development of high-resolution land-use models" (RIKS, 2007a). Metronamica allows for the import and export of possibilities that make it easier to use in conjunction with usually static common commercial GIS software. The package enables the modelling of both supply-side and demand-side forces that drive spatial development using quantitative or qualitative social, economic, demographic, environmental, and physical information at different spatial scales (RIKS, 2005).

3.3 Urban actors

In the applied model, only seven actors are endogenously dynamically modelled. Six of those actors have an external demand (functions), and the area vacant or rural increases or decreases as a result of the total demand for land-use functions. The study focuses on the intraurban allocation of residences at different levels of income. Small services, commerce, and industry are modelled as supporting functions, whose locational decisions are strongly intertwined with residential development. For the case study the actors listed in table 1 are considered.

Table 1. Actors modelled, Metropolitan Region of Belo Horizonte case study.

Actors	
Dynamic	Static
Vacant or rural area	Large industry
High-income residential area	Schools and campuses
Average-income residential area	Water and reservation area
Low-income residential area	Large service and commerce
Small service and commerce	Institutional buildings
Small industry	Airports
In the market (empty cells)	Out of modelling area

3.4 Exogenous features

A few exogenous features were introduced throughout the period studied (1897–2000), including institutional buildings (mainly city halls), airports, university campuses, industrial districts, and major shopping centres, with their exact location and year of construction. The demand of actors (land use) for all other years was estimated on the basis of data from 2000. The spatial distribution for 2000 allocates 5850 (6.3%) cells of 86 m by 86 m to high-income residential areas, 31 672 cells (34%) to average-income areas, and 55 612 cells (59.7%) to low-income areas.

Proportions are maintained and scaled to 1897 data according to the population of each municipality at each time period to obtain the number of cells (demand) per year. This implies constant income distributions throughout the previous century. However, we believe that,

⁽⁵⁾ Licensed by RIKS BV, Maastricht; available at <http://www.riks.nl>

while the structure of the distribution has not changed significantly, the level of income has risen.

3.5 Accessibility

In the proposed model, accessibility to five different types of features is included: (a) planned arterial streets, (b) roads, (c) railway lines, (d) train stations, and (e) metro lines. Each might influence the neighbourhood with a different strength at various distances.

In the application, railway lines are only illustrative and have no impact on actors. Train stations, however, influence locational decisions and function as points of concentration for urban functions. They were added to the simulation at the same location and time of actual implementation. Therefore, in 1897 planned arterial streets, railway lines, and train stations were already in place. In 1951 roads that connect the city to São Paulo, Rio de Janeiro, Brasília, Vitória, northern areas of the state (and Pampulha), and the city's ring road were enhanced or built. In 1965, avenue Cristiano Machado, in the northern part of the city, was added. From 1992 to 2002 the metro line was constructed linking the Eldorado region to Vilarinho, through the central area of the city of Belo Horizonte.

3.6 Dynamics of the case study

The model application, calibration, and validation include comparisons of simulated and real data for various years, and are shown in figure 4.

- (a) The simulation starts in 1897 when planned arterial streets were built. Only 33 known institutional cells are introduced. The aim is to mimic the reference map of 1991.
- (b) The parameters are chosen based on theoretical considerations (discussed in the next section). As part of the calibration, their magnitude is adjusted via changes in the parameters over manual iterative runs of the application from 1897 to 1991.
- (c) For the year 1991, the result of the simulation is compared with actual observations. The decision of whether to accept model parameters as appropriate is based on visual inspection of the resulting map. The accepted parameters are used for projection for the years after 1991.
- (d) The simulation (always starting from the initial year of 1897) continues to 2000.
- (e) Results are then compared with the 2000 validation map.

	1897	1991	2000	2007	2050
Simulation years					
Iterative choice of parameters <i>numerous runs aiming at reference year 1991</i>					
Comparison year 1991 <i>and establishment of parameters as definitive</i>					
Validation I (simulation x actual observations.) <i>actual observation available</i>					
Validation II (simulation x price) <i>price data available (2007)</i>					
Sensitivity analysis					
Scenarios					

Figure 4. Model time scheme and validation.

(f) A sensitivity analysis tests the sensitivity of the neighbourhood rules. The simulation is run again with adjusted parameter settings and results are compared with observed situations for 1991.

(g) Finally, the simulation is run to 2050 to provide general insights into future potential spatial configurations of actors.⁽⁶⁾

3.7 Calibration of neighbourhood rules

The neighbourhood effect discussed in this section falls in the six (I to VI) ‘general rule shapes’ proposed by Hagoort (2006; see figure 5). According to Hagoort, neighbourhood effects might have:

- (a) I and II: a decreasing net positive (or negative) relation that becomes neutral with distance;
- (b) III and IV: a net negative (or positive) relationship that switches to a net positive (or negative) relationship and then becomes neutral; and
- (c) V and VI: an increasing net positive (or negative) relation that switches to a decreasing relationship, and becomes neutral.

Allocation of each general rule to the modelled actors closely follows the proposals of Allen (1997) and Hagoort (2006). Although general rules and shapes can be observed, Hagoort stresses that there are differences of intensity among different regions.

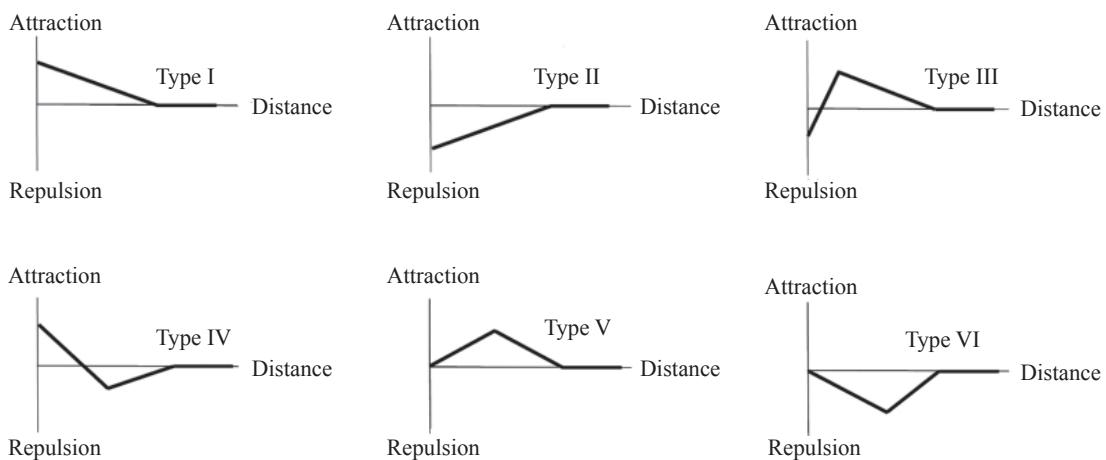


Figure 5. Adaptation of Hagoort’s six general rules (2006, page 69).

The parameters $n_{s(c), s'(c'), d(c, c')}$ depicted in table 2 are chosen iteratively, manually, and, according to the understanding established in the calibration of the model, until the choices that provide the best results (discussed in the next section) are reached. A number of observations can be made regarding the resulting parameters. In general, a small number of rules (represented by parameters) are sufficient for providing reasonable results (discussed in the subsequent section). Consistently, the parameters for high-income areas are higher than values for average-income areas which, in turn, are higher than those for the low-income areas, which implies that preferences are related to income level. The inertia value—at distance 0—for all rules is much higher than the impact of immediately adjacent cells (distance 1).

Rule type I proposed by Hagoort (2006) applies to the impact of residential actors on attractiveness for themselves and the impacts of small service and commerce, large service and commerce, and institutional buildings on residential use—that is, they all have decreasing net positive relationships. Rule type II is suitable for describing the attractiveness of large industry—that is, it has a decreasing net negative relationship.

⁽⁶⁾ Due to size constraints the scenario exercises are not presented here.

Regarding attractiveness between residential actors, both high-income and average-income actors attract each other (type I). However, average-income actors are attracted to high-income actors proportionally more strongly than high-income actors to average-income actors. Proximity to low-income actors has a negative impact on attractiveness for high-income actors (type II). Another rule implies increasing influence starting at further distances. In this case, an actor would like to locate near another but, not in its immediate vicinity.⁽⁷⁾ This is the case for the impact of schools and campuses on residential use.

For impact on small industry, a mix of influences is depicted. The stronger attraction (type I) is for low-income actors; for average-income actors, attraction starts at a greater distance. For high-income actors, attraction is primarily negative but eventually becomes positive (type III). This might indicate a preference for a location which is close to but not in the immediate vicinity of small industry facilities.

Airports, water, and protected natural areas (reservations) are not key elements in this study. The only airport in the city of Belo Horizonte was built along with the entire Pampulha development project. From the 1940s onwards it was intended to be an attraction point. However, this goal was not successfully achieved (Villaça, 1998). Although open space in developed countries (Anderson and West, 2006; Yang and Fujita, 1983), is restricted,⁽⁸⁾ water and natural areas in Belo Horizonte do not have a scenic appeal. However, their occupancy of space is important because it creates a restriction on other actors.

Institutional buildings impact other actors because they are considered central to urban fabric, by attracting other actors to its surrounding regions. For the actor in the market, which represents vacant, speculative land, there is a lack of information such that only the inertia effect is imposed. The influences of other actors (small service and commerce, small industry and in the market) have a supporting role in the modelling of residential actors.⁽⁹⁾

3.8 Calibration of accessibility parameters

The parameters of the accessibility function allow for the influence of multiple types of infrastructure. For the proposed application four types are included. Stations are point locations that refer to actual railway stations. However, more than by facilitating accessibility itself, stations play a role similar to institutional buildings in the sense that they represent a centrality effect. The role of stations as transport nodes is currently very limited because no passenger or freight services are present, except for one 12-hour day trip to Vitoria, a coastal city in the state of Espírito Santo, which has limited importance in Brazil.⁽¹⁰⁾

The other three types of infrastructure are (a) streets originating from the original design that was implemented in 1897, (b) a metro line that was gradually created from 1985 onwards (the power of this line is limited because it is a single line that runs through already densely occupied areas), and (c) roads. These types of infrastructure represent general features of the city. Although they are more specific than a simple variable like distance to CBD, they are not sufficient in providing detailed access information at the level of the neighbourhood.

⁽⁷⁾ Represented with an asterisk (*) in table 2.

⁽⁸⁾ There are only two major parks in the city of Belo Horizonte. One is centrally located within a heavily commercial service area and is considered popular (in a sense of mass consumption, not sophisticated or exclusive). It is close to a polluted, fetid river. The other has exclusive access, with an entrance at the end of a high-income area. Other green areas, such as Parque Lagoa do Nado, are missing infrastructure or maintenance.

⁽⁹⁾ The results table can be requested from the authors.

⁽¹⁰⁾ Most intraurban transportation consists of buses or private vehicles (Gouvêa, 2005) Large freight mining companies, such as Vale, use private railway lines.

Table 2. Parameters of neighbourhood influence ($n_{s(c), s'(c'), d(c, c')}$).

Distance (in cells)	$d(c, c')$												
	0.00	1.00	1.41	2.00	2.24	2.83	3.00	3.16	3.61	4.00	4.12	4.24	4.47
<i>High-income</i>													
High-income	95.00	14.00	5.96	1.00	0.96	0.86	0.83	0.81	0.73	0.67	0.65	0.63	0.59
Average-income	17.92	5.21											
Low-income	-5.00	-1.65											
Small service and commerce	14.00	4.30	1.84	1.68	1.61	1.45	1.40	1.35	1.23	1.12	1.08	1.05	0.90
Small industry	-2.83	-1.08	-0.35	0.68	0.93	0.83	0.81	0.78	0.71	0.65	0.63	0.61	0.57
In the market													
Large industry	-1.00	-0.50	-0.29										
Schools and campuses													
Water and reservation areas													
Large service and commerce	14.00	12.25	11.53	10.50	10.09	9.05	8.75	8.47	7.69	7.00	6.78	6.58	6.17
Institutional buildings	12.00	9.75	8.82	7.50	6.97	5.64	5.25	4.88	3.89	3.00	2.72	2.45	1.94
Airports													
<i>Average-income</i>													
High-income	55.66	6.96											
Average-income	85.85	14.00	3.97										
Low-income													
Small service and commerce	5.90	2.20	0.67										
Small industry													
In the market													
Large industry	-0.66	-0.33	-0.19										
Schools and campuses													
Water and reservation areas													
Large service and commerce	9.70	4.85	2.84										
Institutional buildings	20.75	7.52	2.04										
Airports													
<i>Low-income</i>													
High-income													
Average-income													
Low-income	67.00	0.55	0.32										
Small service and commerce	1.20	0.60	0.35										
Small industry													
In the market	3.77	2.36	1.77	0.94	0.61								
Large industry	-0.08	-0.04	-0.02										
Schools and campuses													
Water and reservation areas													
Large service and commerce	3.00	1.50	0.88										
Institutional buildings	20.75	16.86	15.25	12.97	12.05	9.75	9.08	8.45	6.72	5.19	4.71	4.24	3.35
Airports													

Further detailing this accessibility information with a historical study of their implementation would provide better results, but it would also hamper generalization of the results to other case studies.

The parameters [see equations (5) and (6) and table 3] are chosen such that:

- (a) For high-income actors, the most important factor is the ‘formally’ planned city represented by the original central ring that had acquired the status of capital or political centre (outer localities were deemed rural, suburban areas). Stations are also important for municipalities other than Belo Horizonte. Metro lines are less important, given the ownership of cars by most households in high-income areas. Roads, although important, are responsible for intercity connections, rather than intraurban connections that are the focus of this study.

																	Hagoort rule type
5.00	5.10	5.39	5.66	5.83	6.00	6.80	6.32	6.40	6.71	7.00	7.70	7.21	7.28	7.62	7.91	8.00	
0.50	0.48	0.44	0.39	0.36	0.33	0.32	0.28	0.27	0.22	0.17	0.15	0.13	0.12	0.06	0.03		I
0.84	0.81	0.73	0.65	0.61	0.58	0.54	0.47	0.45	0.36	0.28	0.26	0.22	0.20	0.11	0.05		I
0.48	0.47	0.42	0.38	0.35	0.32	0.31	0.27	0.26	0.21	0.16	0.15	0.13	0.12	0.06	0.03		III
														0.70	0.10	0.30	II
																	*
5.25	5.08	4.58	4.10	3.80	3.50	3.36	2.93	2.79	2.26	1.75	1.63	1.38	1.26	0.67	0.33		I
0.75	0.53																I
																	I
																	I
																	I
														0.20	0.43	1.55	2.20
															2.83		*
														0.01	0.03	0.05	II
																	*
																	I
																	I
																	I
																	I
																	I
																	I
																	I
1.30	0.91																I

*—See text for explanation.

- (b) Average-income actors value both planned streets and stations that give them access to the city.
- (c) Low-income actors value accessibility of stations the most, followed by the metro line, and then the original planned streets.

3.9 Calibration of land-price parameters

The first stage is to calculate prices of land based on the actual configuration of actors. The impact of each actor on prices is given by parameters (μ_s), as depicted in table 4. When calculating the transition potential for the next time step, the affordability of different actors concerning land prices is weighted by τ_s , as explained in section 2.3.

Table 3. Parameters of accessibility: distance decay ($a_{y,s}$) and relative importance ($w_{y,s}$).

	Stations	Railway	Planned street and arterial corridors	Metro	Roads
<i>High-income residential area</i>					
Distance decay	10	0	10	10	20
Relative importance	0.4	0	0.9	0.05	0.01
<i>Average-income residential area</i>					
Distance decay	10	0	50	10	20
Relative importance	0.32	0	0.29	0.1	0.05
<i>Low-income residential area</i>					
Distance decay	5	0	15	20	10
Relative importance	0.25	0	0.15	0.2	0.01
<i>Small service and commerce</i>					
Distance decay	10	0	10	10	10
Relative importance	0.4	0	0.8	0.2	0.1
<i>Small industry</i>					
Distance decay	8	3	2	6	6
Relative importance	0.7	0	0.2	0.1	0.4
<i>In the market</i>					
Distance decay	30	0	100	30	100
Relative importance	0.35	0	0.4	0.2	0.2

Table 4. Parameters used for prices.

Function (s)	μ_s	τ_s
High-income	8	0.1
Average-income	3.2	0.24
Low-income	0.8	0.95
Small service and commerce	10	0.05
Small industry	2	0.3
In the market	8	0.24

The parameters are chosen to have as reference the relative difference in price obtained empirically for the city of Belo Horizonte and also based on data by Furtado (2007). The radius used for the calculation of $[D(c)]$ is 4 cells.

4 Results and validation

The results of the simulation, using the above parameters, provided yearly results presented as maps on a ten-year time interval, starting at 1900 (figure 6).⁽¹¹⁾ Figure 7 presents the results for 1991 in detail. Figure 8 presents the empirical observed results for comparison.

Given the influence of the chosen parameters, the allocation of actors is scattered throughout the study area and is polycentric in the initial years. However, the model projects clustering near the planned area. Comparison with actual initial development shows that in the simulation results the central ring is occupied earlier. As discussed in the presentation of the model mechanisms, it is rather difficult to simultaneously simulate attraction and repulsion.

⁽¹¹⁾Because of the size, see the legend in figure 7.

As time unfolds, however, the pattern of occupation observed in the simulation is similar to the actual pattern of occupation.

The change starting in the 1950s captures the expansion in the number of inhabitants.⁽¹²⁾ These years also show the expansion to the west of mainly average-income residential areas, which the model realistically simulates. The clustering of high-income actors in the centre, surrounded by a large patch of average-income actors' occupation, along with external, more dispersed, conurbated low-income residential areas, is similar to the actual, observed pattern.

Table 5 shows that the general structure depicted in the empirical reference map of 1991 (on the left-hand part of the table) is present in the map of the simulation results (on the right-hand part of the table).⁽¹³⁾ The high-income fractal dimension in the simulation is close to observed data, but with a slightly higher patch size, which indicates that the simulation map is more clustered than the actual results. For average-income actors, patch size numbers of the simulation are close to reality, but the perimeter is a bit lower in the simulation results, which suggests a lower degree of clustering than in the real data. The low-income configuration seems to differ more from reality than the others because the patch size of the simulated results is much larger than the one observed. This suggests that low-income actors are more clustered in the simulation than they should have been.

4.1 Validation

The transition between 1991 and 2000 presents the conurbation of low-income areas to the north and a strong expansion of high-income actors towards the southern municipality of Nova Lima. This reflects the construction of the first major shopping centre in the late 1970s and a continuous preference of residential actors for the southern region (Costa et al, 2006; Villaça, 1998) (see figures 9 and 10).

In relation to the reference map of 2000, the metrics (table 6) indicate that the simulation performs well and that the similarity between simulated and observed land-use patterns for 2000 is larger than for 1991. Apart from the patch size and perimeter of average-income areas, all indicators remain close to the actual data of 2000. This suggests that the model that is calibrated on the 1897–1991 period with the chosen parameters settings is valid—that is, the model is able to reproduce observed spatial patterns beyond the calibration period (Batty and Torrens, 2005).

4.2 Sensitivity analysis

The sensitivity analysis is reported in this section. Four different scenarios are tested and detailed below: (a) in the first, there is no influence from prices, so that equation (1) becomes: $T_{s,c} = [(1 + e)^t N_{s,c} A_{s,c}^{\text{total}}]$; (b) in the second, the influence of prices, given by parameter τ_s , is doubled; (c) the third test applied is to run the test with accessibility influence set to zero; and (d) a fourth test observed is to neutralize the income-differentiated actors proposed. This is applied by equalizing the parameters of table 2.

⁽¹²⁾This information is exogenously supplied to the model, which should capture the locational choice of newcomers rather than just their aggregate numbers.

⁽¹³⁾The metrics used to compare the maps are described as follows. Most metrics are based on patch size. Patches are “groups of contiguous cells that are taken in by the same category” (Hagen-Zanker, 2006). For each patch, its size, perimeter, and edge length can be measured. Other common (and popular) descriptive numbers are fractal dimension and the shape index (Batty, 2005b; Batty and Longley, 1994; Benenson and Torrens, 2004). Fractal dimensions provide a single number for a map which indicates how much of space is completely filled with a certain state or category. Two, for instance, represents a square filled with the same state, class or category. The shape index, in turn, is calculated as the perimeter divided by the square root of the patch size. It is easy to see that “larger values indicate a more convoluted shape” (RIKS, 2006).

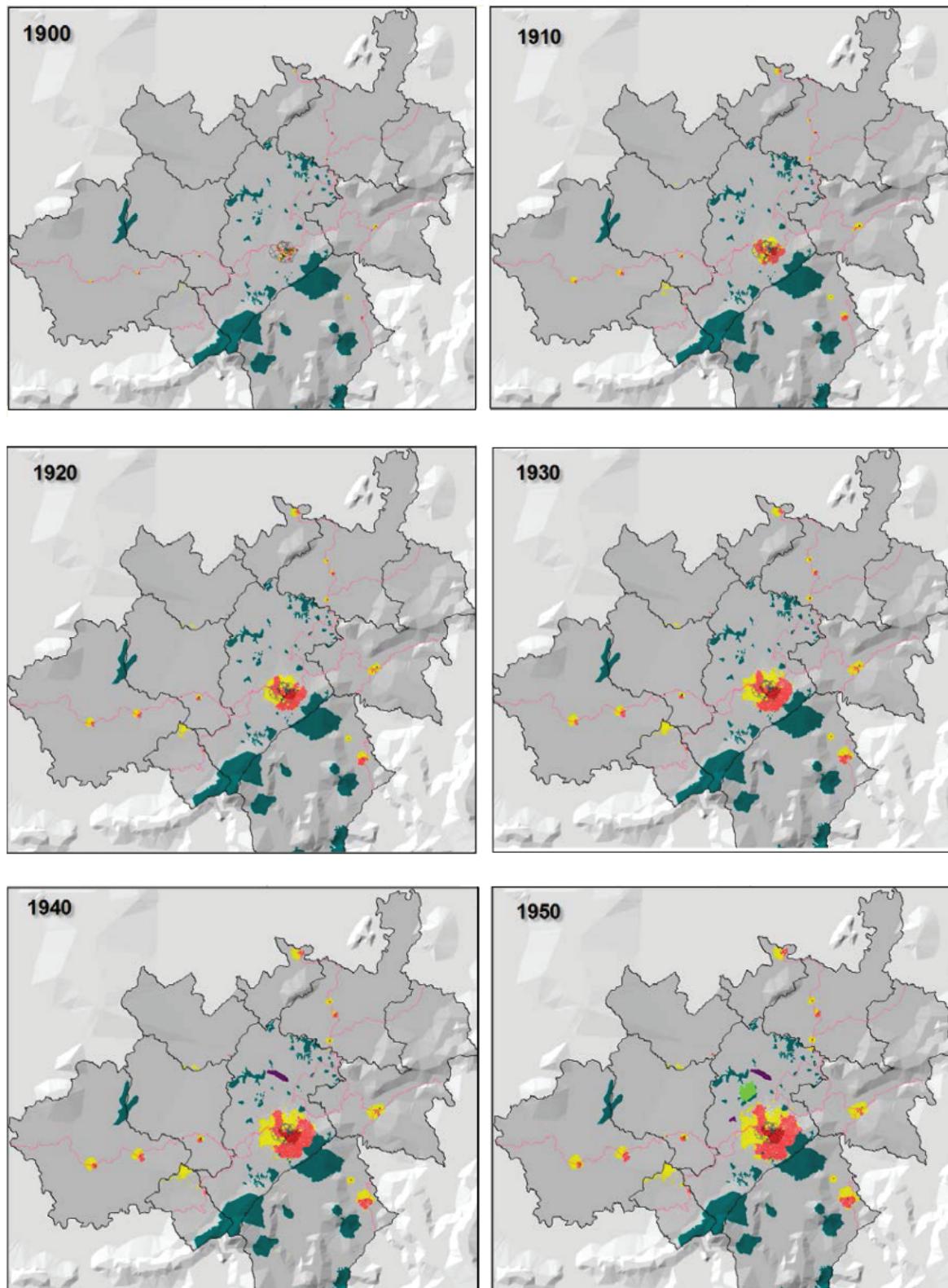


Figure 6. [In colour online.] Evolution of actors in the Metropolitan Region of Belo Horizonte, every ten years (1900–90).

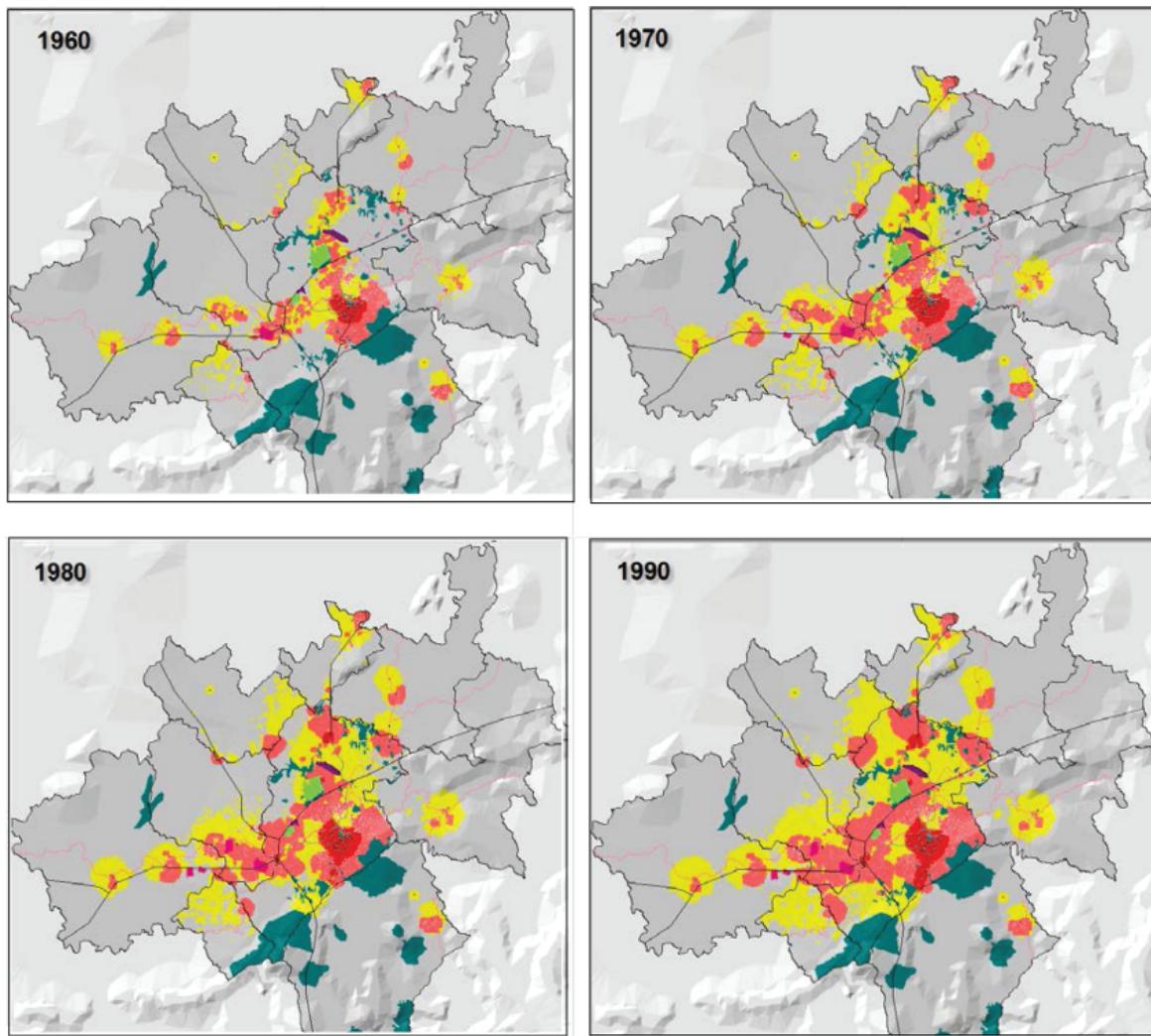


Figure 6 (continued).

Table 5. Calibration results: comparison of basic measures of the reference map with the simulated map for 1991.

Basic measures of actual reference map 1991				Basic measures of simulation map 1991			
high-income	average-income	low-income	global	high-income	average-income	low-income	global
<i>Fractal dimension</i>							
1.327	1.403	1.343	1.364	1.313	1.372	1.413	1.394
<i>Patch size</i>							
1863	8594	4229	5712	2338	8377	6868	7215
<i>Perimeter</i>							
641	2900	1168	1775	654	2442	2212	2227
<i>Shape index</i>							
3.547	6.941	4.326	5.242	3.342	6.021	6.300	6.067

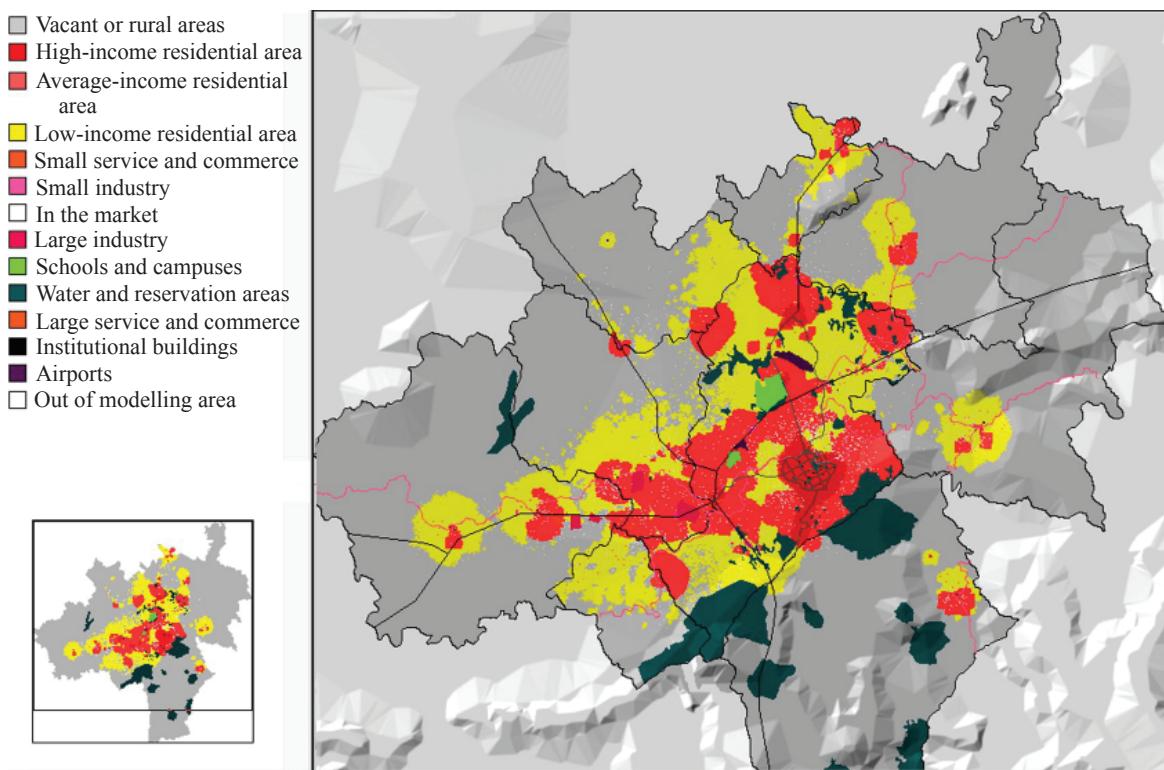


Figure 7. [In colour online.] Simulated results for 1991.

Table 6. Validation results: comparison of basic measures of the reference map with the simulated map for 2000.

Basic measures of actual reference map 2000				Basic measures of simulation map 2000			
high-income	average-income	low-income	global	high-income	average-income	low-income	global
<i>Fractal dimension</i>							
1.334	1.429	1.386	1.397	1.348	1.370	1.439	1.410
<i>Patch size</i>							
1863	14 290	12 819	12 624	3079	8695	12 554	10 646
<i>Perimeter</i>							
669	4985	3294	3651	906	2412	3958	3240
<i>Shape index</i>							
3.547	9.246	6.563	7.294	3.972	5.857	8.033	7.037

The results of the simulation in which changes in price are not implemented (figure 11) suggest that the effect of the neighbourhood influence is stronger than that of the land prices as implemented. However, the edges and borders of the simulation without prices are more compact, regular, and do not resemble the empirical data (see figure 8).

In the second analysis the influence of prices, reflected by the values for τ_s , is doubled (figure 12). As expected, the stronger the impact of price, the stronger the effect of succession and a larger part of the low-income residential area is expelled to the outskirts of the conurbation patch. This happens because the value of τ is smaller for high-income actors and higher for low-income actors. However, even with these strong impacts of price, the inertia effect of some low-income residential areas is strong enough to keep them in central

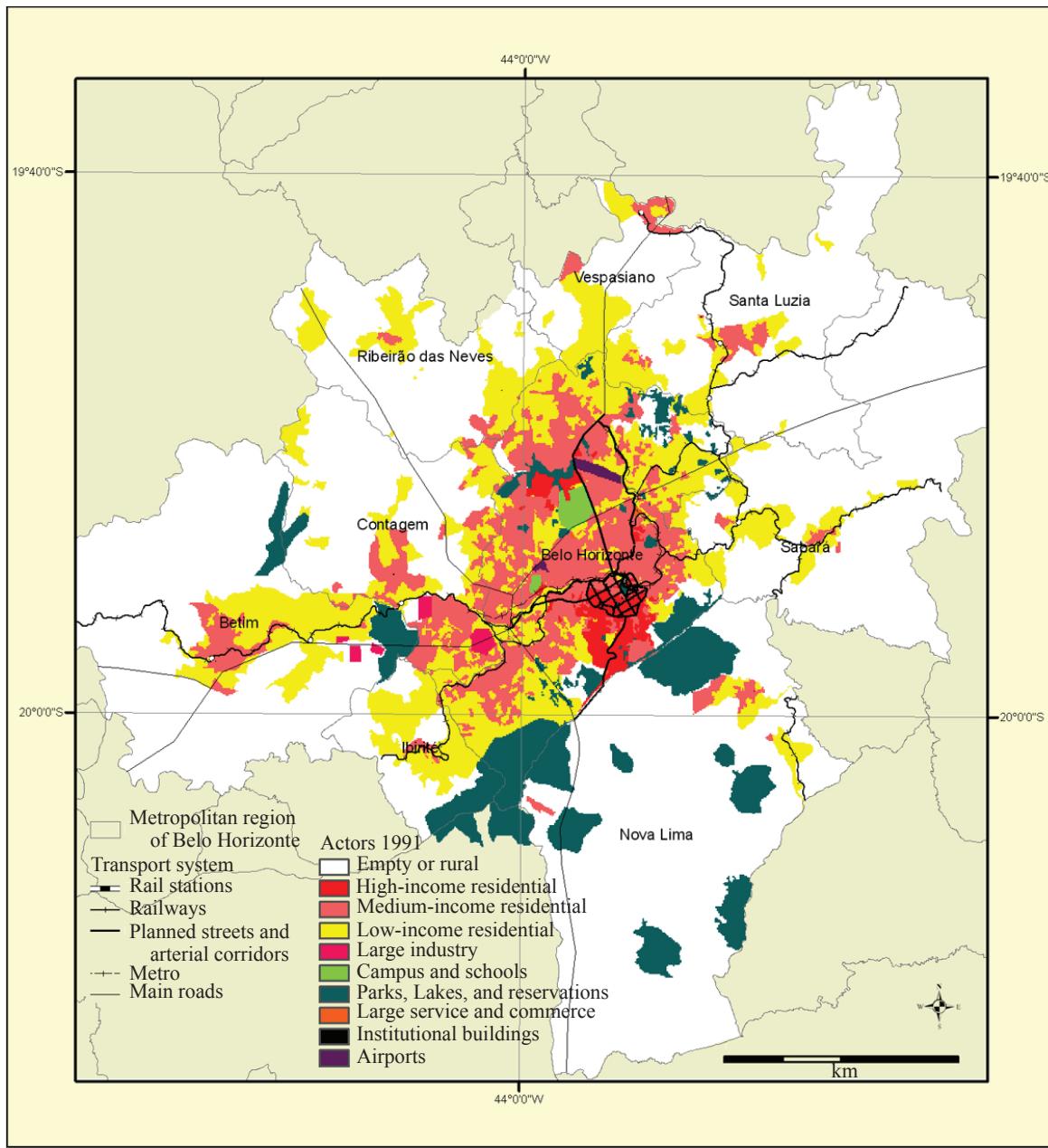


Figure 8. [In colour online.] Empirical configuration of actors, 1991 (adapted from Brazilian Institute of Geography and Statistics).

areas, and a smaller portion of them do so. Apart from that and a more convoluted edge, there are no other significant differences.

Third, simulation results without accessibility parameters show the importance of the centrality represented by these parameters (figure 13). This modification of the model gives results that are very different from the observed pattern. Instead of presenting a large patch of low-income residential areas attached to central high-income and average-income areas, it is presented as a number of smaller patches in an archipelago-like configuration. This is also true for the average-income area that is much more dispersed. For the high-income residential areas, the configuration is similar, but its expansion is more irregular and does not start to unfold towards the southern vector as was observed in reality. The impact of institutional buildings and their overpowering of both neighbourhood parameters and price affordability seem to restrict allocation to the central, planned area.

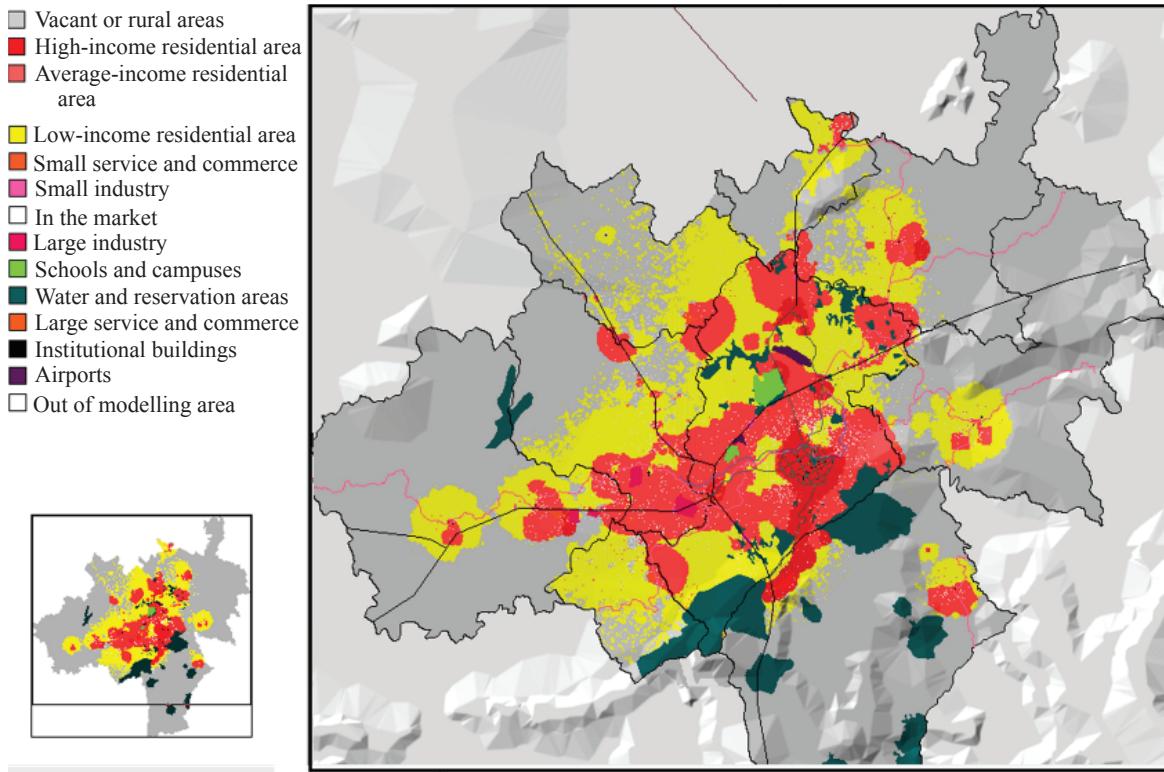


Figure 9. Simulated results for 2000.

Fourth, in order to evaluate the adequacy of having differentiated actors the chosen parameters are set to have similar parameters for all residential actors (figure 14). Implicit in this test is the homogeneous treatment of residential actors, which is often the case in other CA models. The results demonstrate the importance of explicitly accounting for social heterogeneity. The pattern observed in this exercise is rather different from patterns in figure 7. The high-income residential area becomes too attached to the accessibility network, especially to 'roads'. Average-income areas do not seem to conform to any regular pattern, and are subdivided into a larger number of patches. The low-income areas are more clustered than they should be, although they do not form one large external patch.

The sensitivity analysis suggests that neighbourhood influence is central to the spatial configuration of the city, and prices play an important role in the dynamics of this influence. That is, the higher the influence of prices, the faster is the segregation and succession among different income-level neighbours. Inertia—the tendency of earlier occupants to remain for long periods of time in a given area—is the countereffect of succession. Accessibility is also an essential item because it organizes the concentration of actors in the city. Finally, results show that an intraurban analysis cannot fail in differentiating actors by income level. Exercises that include only urban versus nonurban land use are reasonable when studying urban morphology as a whole, but not when studying its internal configuration.

5 Concluding remarks

The proposed model simulates the dynamics of urban development by explicitly distinguishing between positive and negative forces of agglomeration. By doing so, it goes beyond the model proposed by White and Engelen (1993) and the subsequent models, in which all effects of local influence on urban development are modelled in an aggregated manner. The sensitivity analysis indicates that modelling heterogeneity and land-prices feedback can influence significantly the results of the model.

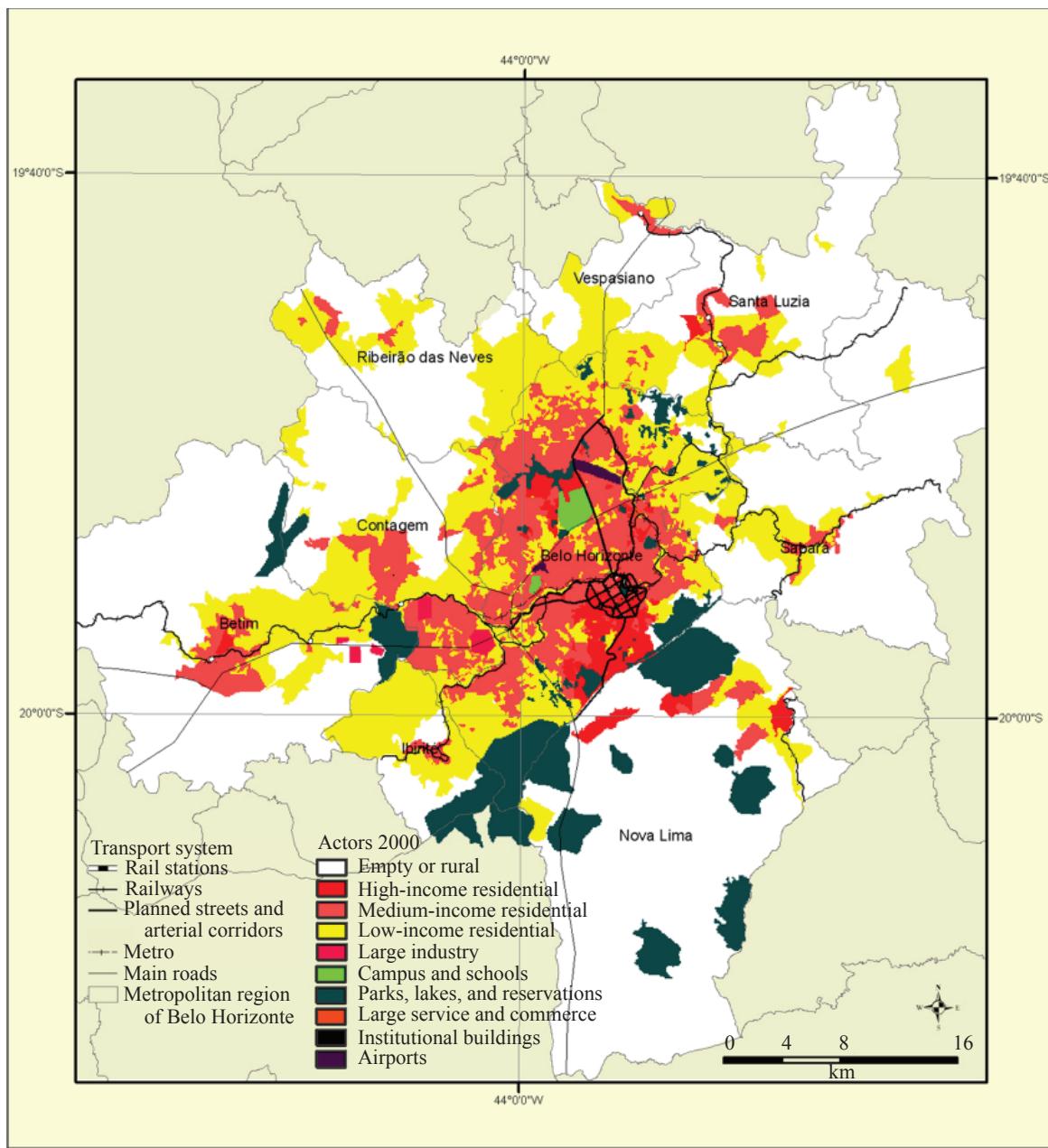


Figure 10. Empirical configuration of actors, 2000 (adapted from Brazilian Institute of Geography and Statistics).

Additionally, the paper shows that a developing urban area can be simulated by using long-term feedback processes in a rather large area, even if this means that the number of actors involved increases considerably. The results indicate that the dynamic influence of relative prices and interactions among actors is important in reaching a comparable result.

Specifically, sensitivity analysis shows that neighbourhood influences, or the attraction forces observed among similar and different actors in a neighbourhood, are an important issue in explaining urban dynamics for the case study. Spatial heterogeneity also plays an essential role, because actors entering the system weigh possible location alternatives in a nonlinear fashion. This results in a multimodal, complex configuration that is in harmony with the empirically observed distribution. Social heterogeneity is a crucial element as well. As the sensitivity analysis demonstrates, neglecting the socially diverse dimension of actors yields an unsatisfying result for the case study.

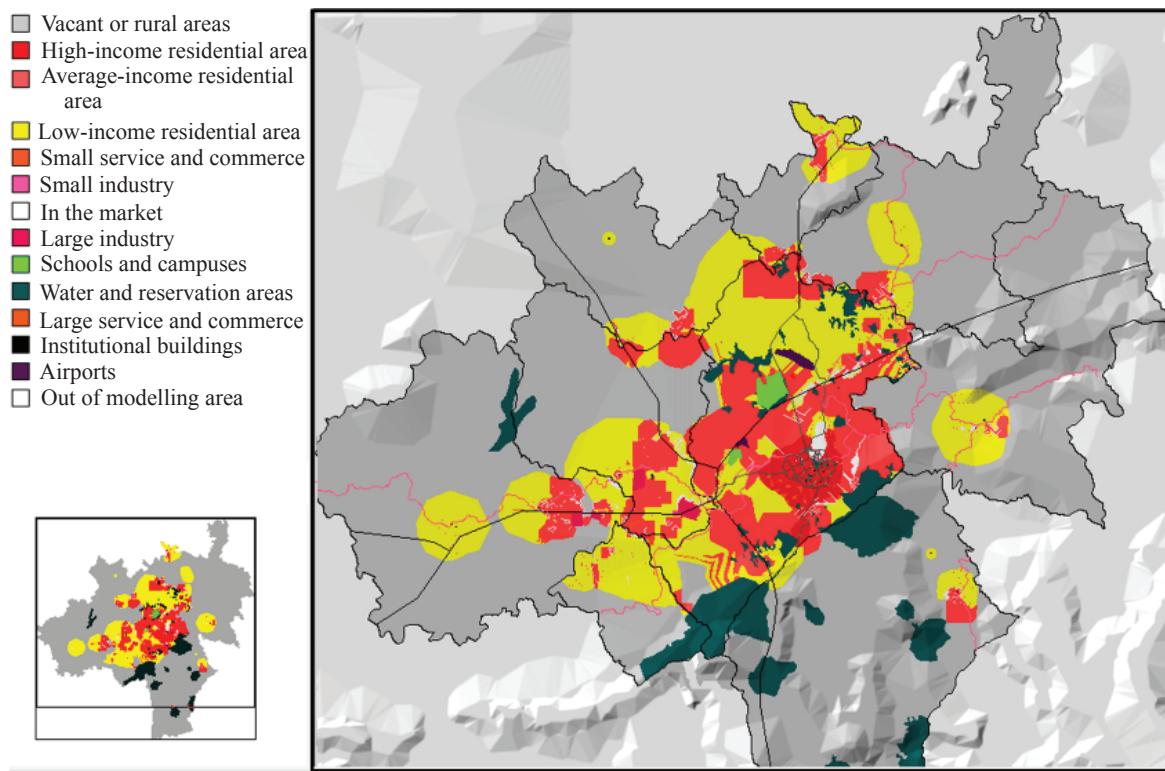


Figure 11. [In colour online.] Results of the simulation without price changes, 1991.

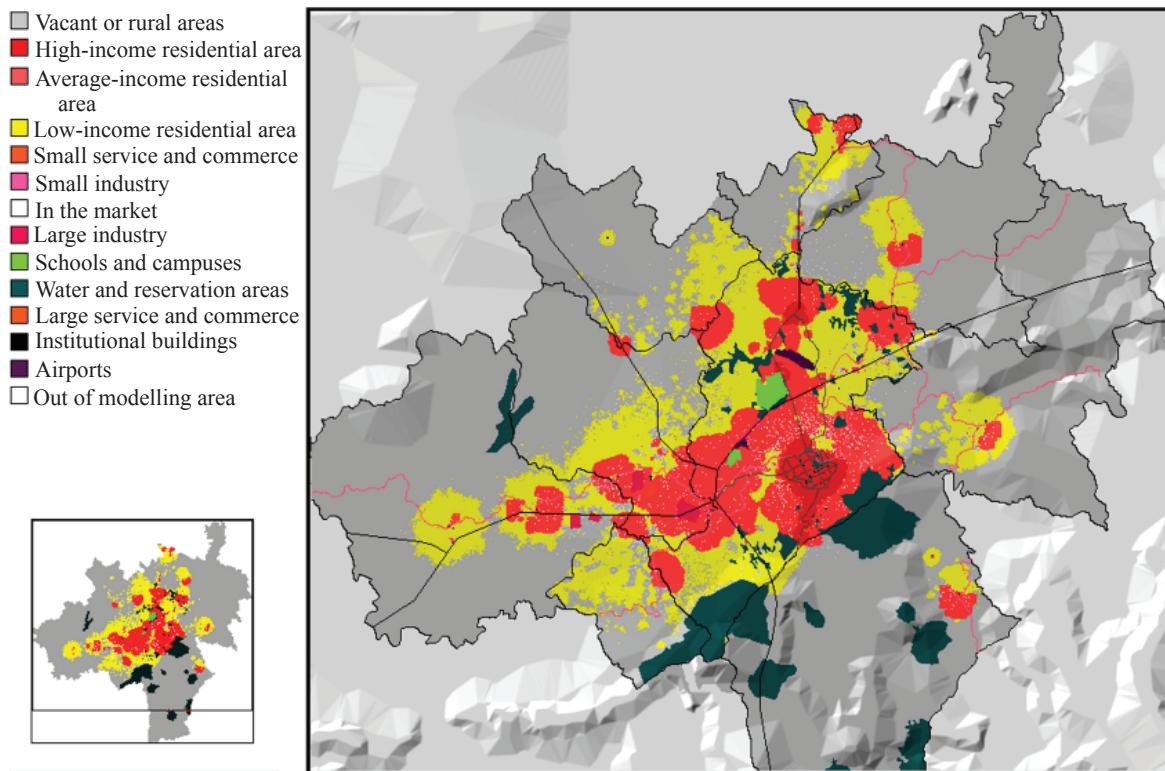


Figure 12. [In colour online.] Simulation results with double influence of prices, 1991.

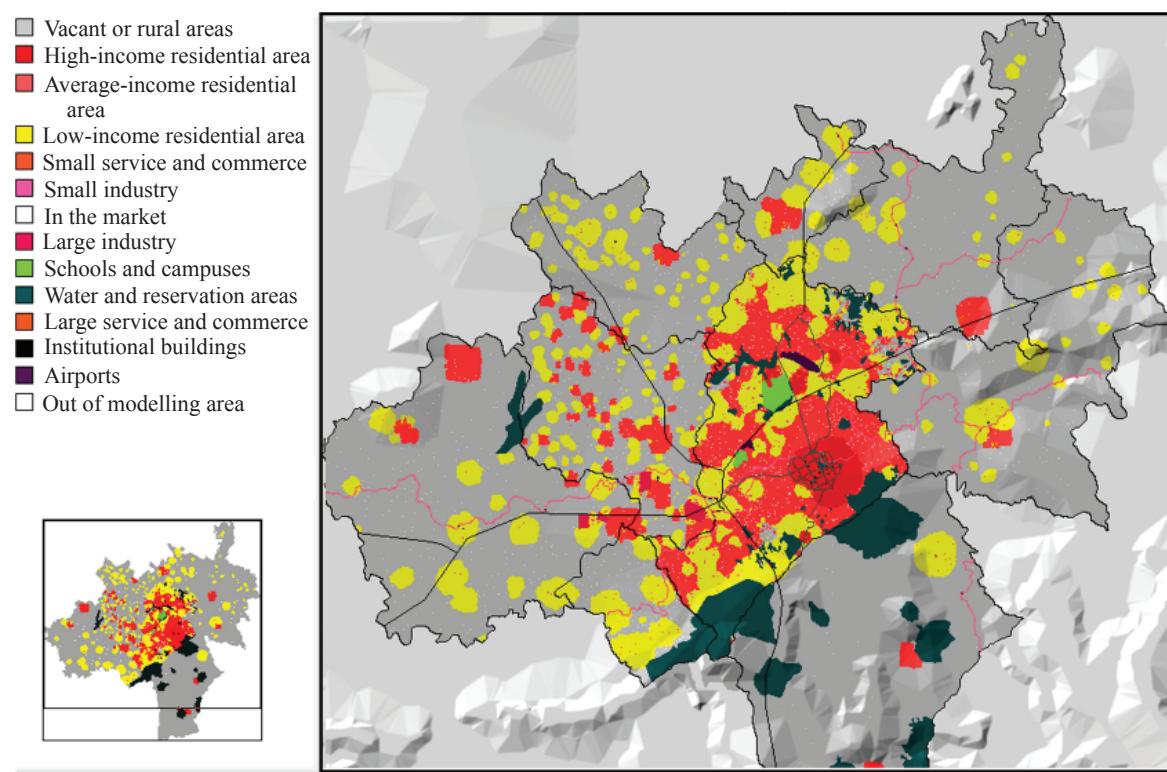


Figure 13. Simulation results without accessibility parameters, 1991.

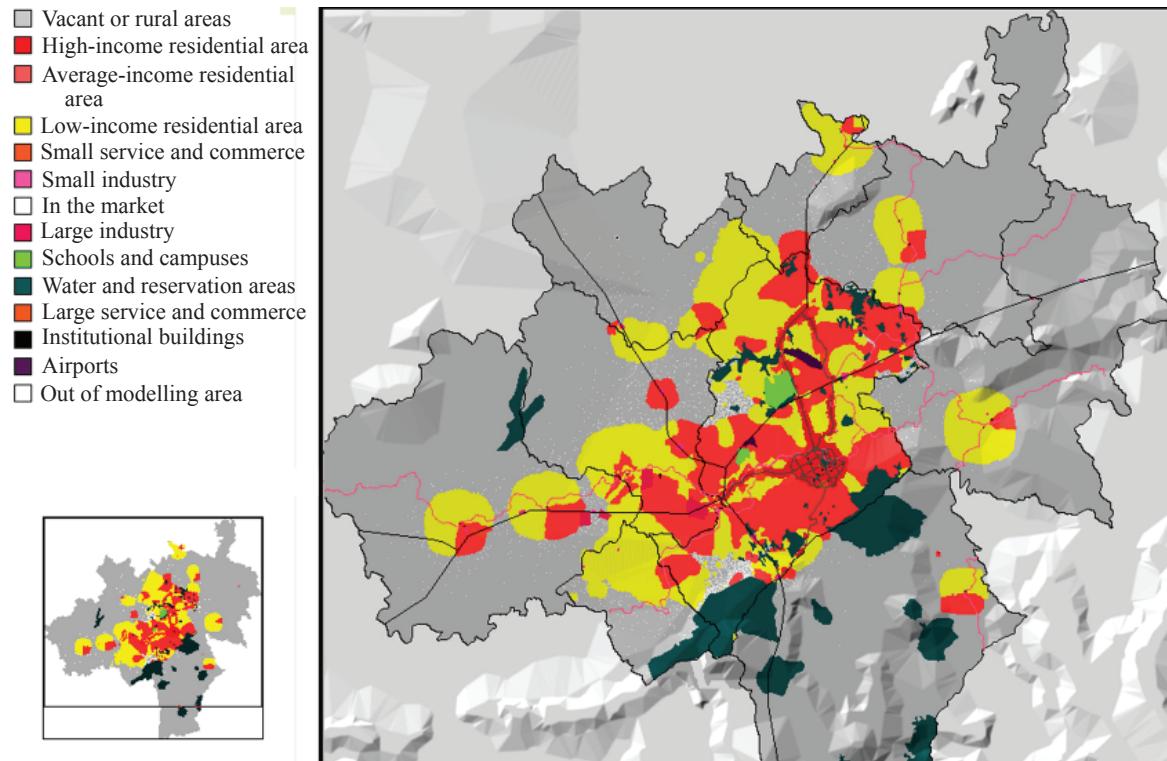


Figure 14. Simulation results with similar N parameters for all residential income levels, 1991.

Although exerting a repulsion effect for all actors, land prices impact those with less financial power more strongly. The parameters of the model provide some insight into the intensity of the different impacts upon each actor. Further, the proposed extended model ratifies the fact that any intraurban analysis should differentiate urban actors by income level as their preferences and financial constraints are determinant to urban spatial configuration.

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