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A didactic note on the use of Benford's Law in public works auditing, with an application to the construction of Brazilian Amazon Arena 2014 World Cup soccer stadium

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A didactic note on the use of Benford's Law in public works auditing, with an application to the construction of Brazilian Amazon Arena 2014 World Cup soccer stadium[1]

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Abstract

Globalization requires developing countries' governments to invest heavily in expensive large-scale infrastructure projects in order to keep on the map of an ever more competitive world. In a context of capital constraint, it is essential to keep public procurement works at their lowest possible cost while assuring a high quality output. This paper introduces Benford's Law as a tool to detect overpricing in worksheets of public works. That law suggests that the frequency of the first digit in a multitude of non-manipulated numerical databases decreases successively from digit 1 (about 30%) to digit 9 (less than 5%). The paper describes a few relevant statistical tests of Benford's Law and applies them to the construction work of Brazil's Amazon Arena 2014 World Cup soccer stadium. Then, it compares Benford's Law results with those obtained from the analysis of prices conducted by the Brazilian Court of Accounts (TCU). The tests identified items in the worksheet that did not comply with the Law and corresponded to over 80% of the total overprice uncovered by TCU. That identification required auditing only 65% of total procurement costs, whereas the methodology used by TCU audited about 80% of total costs. Finally, we propose an alternative algorithm for selection of the sample to be audited while still auditing the conventional 80% of total costs.

Keywords: Audit. Public works. Benford's Law. Overpricing.

1. Introduction

According to Japan Statistics Bureau, in 2010 the country had 1751 municipalities[2] where lived a total population of 128,057,352 people[3]. Consider a database composed of each one of the cities' populations; then calculate the number of cities whose population count's first digit is 1, such as the city of Kyoto, 1,474,015 inhabitants or Tarumisu, in Kagoshima, with 16,702 inhabitants. Do the same for all other possible first digits, from 2 to 9. What should we expect the relative frequencies of the number of cities in each one of these 9 categories to be?

A natural guess would be that, a population of a city being a random number, each category would contain approximately the same number of cities, i.e., each relative frequency would be roughly 1/9. Figure 1 below suggests that the naïve guess maybe quite incorrect.

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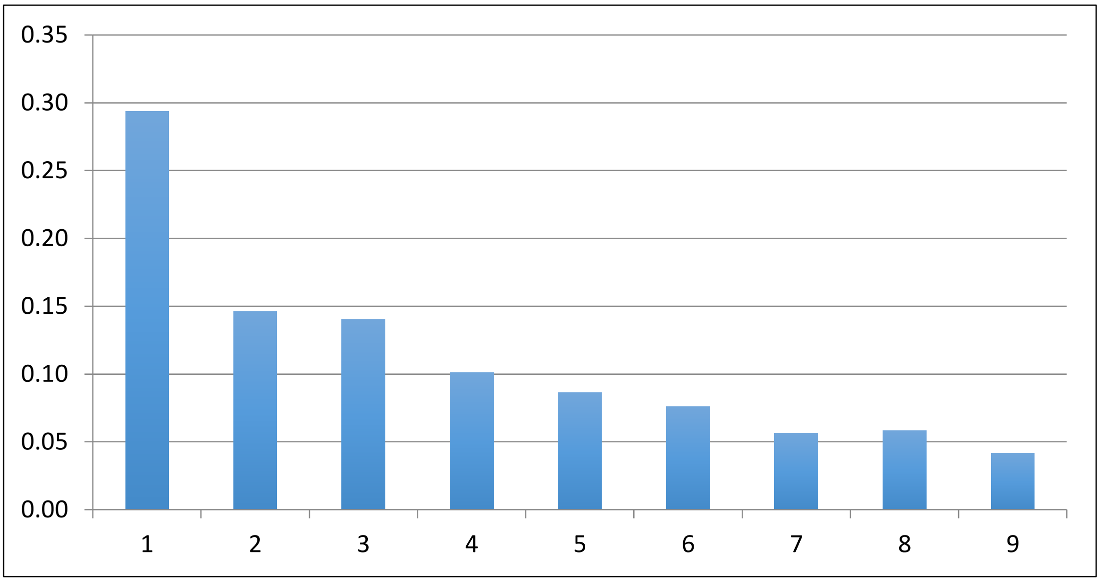
[1] The authors are grateful to Brian Gaines for introducing Benford's Law and to Adriana Portugal most especially to Keiichi Yamazaki for discussions, comments and encouragements. The authors remain sole responsible for opinions expressed, remaining errors or omissions. Mauricio Bugarin gratefully acknowledges the Brazilian National Research Council, CNPq, research grant.

[2] We define municipality here as an area where election for mayor takes place. Japanese municipalities are classified as cities (市 , 790), special wards of Tokyo (区 *ku*, 23), towns (町 *machi*, 745) and villages (村 *mura*, 183), the 市区町村 *shikuchoson* system.

[3] http://www.stat.go.jp. Accessed May 21, 2015.

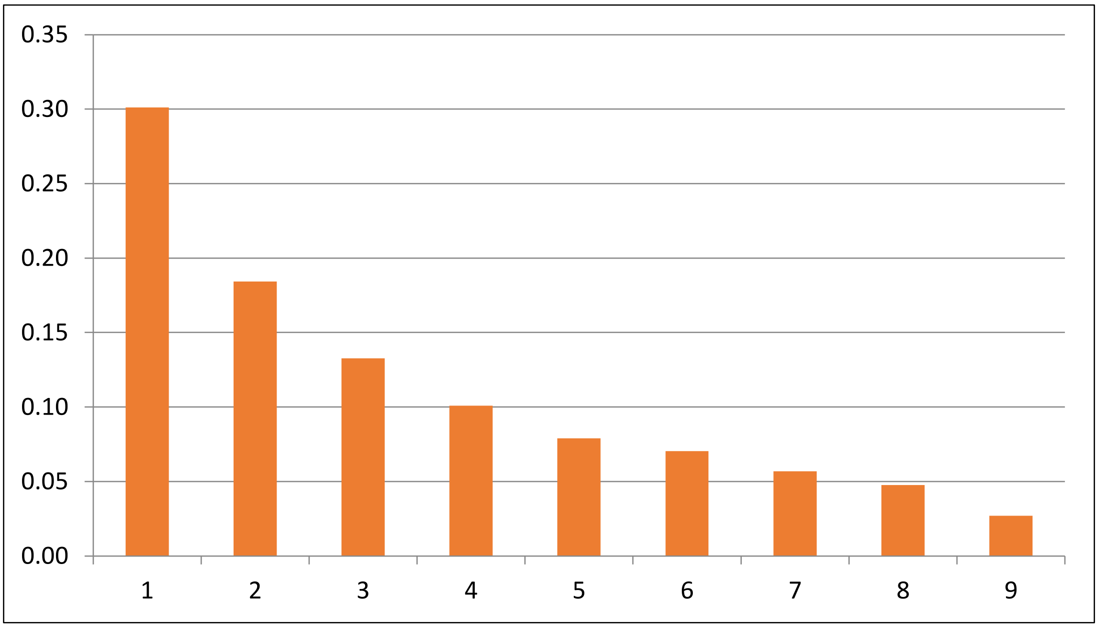
Indeed, Figure 1 hints that the percentage of Japanese cities whose population counts has first digit *i* decreases from almost 30% to less than 5% as *i* increases from 1 to 9. Figure 2 presents the corresponding graph for the Brazilian municipalities' populations in 2010, showing a similar pattern. In fact, the observed first-digit-decreasing-frequency appears to be a rather general property of databases collected from natural, non-manipulated sources.

Figure 1 – Relative frequencies of first digits in Japan city populations in 2010.



Source: Local Administration Bureau, Ministry of Internal Affairs and Communications, Japan

Figure 2 – Relative frequencies of first digits in Brazilian municipalities populations in 2010.



Source; Brazilian Institute of Geography and Statistics, IBGE.

It was probably 19th century mathematician Simon Newcomb (1881) who first grasped the first-digit-decreasing-frequency stylized-fact when he noticed that the first pages of logarithm tables were more worn than the following ones, suggesting that the most commonly accessed value was 1 (Newcomb, 1881). Newcomb actually suggested the correct mathematical expression for the corresponding distribution, but he did not gather numerical data or provide other evidences supporting his claim. His work remained little known until, over half a century later, physicist Frank Benford (1938) reached the same conclusion, apparently independently but also motivated by observation of the wear of logarithm tables. Benford published a seminal article in 1938, “The Law of Anomalous Numbers”, which used data collected from numerous different sources. These data were random, and not related to each other, and ranged from numbers collected from the front pages of newspapers to river lengths and to mathematical tables and scientific constants. He recorded the first digit of the collected data and found that: 30.6% of the numbers had 1 as the first digit; the first digit 2 occurred in 18.5% of cases; and so on, in a decreasing manner, until first digit 9, which corresponded to only 4.7% of the numbers in the database. Such frequencies of first digits were confirmed to appear in a variety of databases, including energy bills, addresses, stock prices, city population values, and mortality rates, among others. That distribution is known as **Benford distribution** and the property discovered by Newcomb and Benford is known as **Newcomb-Benford’s Law** or, more simply, **Benford’s Law**.

In order to better understand the differences in frequency of the first digit, suppose you invest 10,000.00 dollars in an investment bank that assures you a fixed return of 7% per annum. Then, your investment will double roughly every ten years. Therefore, after ten years with 1 as the first digit, the balance of your investment will eventually reach 20,000.00 dollars. After another 10 years, the balance will double to 40,000.00, so the numbers 2 (first part of the decade and 3 (second part) will appear in 10 years. After another decade, the amount will reach 80,000.00, so that the digits 4, 5, 6 and 7 will successively appear as first digits in the ten-year period. Eventually, the investment will reach the value of 100,000.00, with the first digit 1 materializing for another ten years, and so on. Thus, when choosing a random date, it is more likely that the balance of your investment’s first digit is 1 than any other digit.

A database is more likely to follow a Benford distribution when data are collected from different sources (Hill, 1995) or when “the elements result from random variables taken divergent sources that have been multiplied, divided, or raised to integer powers” (Durtschi et al., 2004). In particular, construction works procurement data, which involve quantities (items to be used in the construction) multiplied by prices (unit prices of these items), which come from different distributions, seem particularly fit to follow Benford’s Law. Furthermore, the larger the database, the more likely it will conform to the Benford’s Law.

On the other hand, numbers assigned by human intervention, such as Social Security numbers, postal codes, bank accounts, phone numbers, or numbers produced by students in experiments usually do not conform to Benford’s Law (Nigrini, 2000). This observation suggests that the “Law of Anomalous Numbers” may be used to detect evidence of human manipulation of data. Indeed, by altering the original data, one will most likely create a new distribution that does not conform to Benford’s Law.

Naturally, deviations from Benford’s distribution do not constitute conclusive proof of manipulation, just as compliance does not ensure data reliability. However, nonconformity can be seen as a signal that the data need scrutiny. Thus, Benford’s Law (NB Law) can be used in conjunction with other control mechanisms as a guide to check for possible manipulations.

The literature presents several empirical analyses based on the hypothesis that fabricated data do not follow Benford’s distribution. Nigrini (2012), assuming that true financial data followed Benford distribution closely (as indicated by his previous research), argues that substantial deviations from this law suggest possible fraud or concocted data. Nigrini developed several tests to measure compliance with Benford's Law, and the Wall Street Journal (Berton, 1995) reported that the Attorney's office in Brooklyn, New York, detected fraud in seven companies in New York using these tests. The evidence found that fraudulent data reported too small frequencies of first digit 1 and too high frequencies of first digit 6. Based on the success cases, Nigrini became a consultant to internal revenue agencies of different countries and developed computer tests of NB Law to detect fraud that are currently being used by those agencies.

Göttsche, Brähler and Engel (2011) use Benford's Law to discuss evidence of manipulation in macroeconomic data, and suggest which data needs a more rigorous inspection. The paper studied the first digit of macroeconomic data reported to the Statistical Office of the European Union (Eurostat) for EU countries and constructed a ranking of the 27 member countries according to the extent of the deviation from NB Law predictions. The country that presented the highest deviation was Greece, which manipulation of the data had been officially confirmed by the European Commission (2010).

Cho and Gaines (2007) analyze in-kind contributions to joint fundraising committees in six successive federal electoral campaigns in the US, from 1994 to 2004. The authors find that the committee-to-committee, in-kind contributions catalogued by the US Federal Elections Commission (FEC) show an increasing non-compliance with Benford's Law, which could be interpreted as a higher degree of electoral campaign data manipulation in more recent election cycles as opposed to older ones.

University of Michigan professor Walter Mebane analyzed election data from several countries and discovered that the count of votes tended to follow Benford's Law for the second digit (Mebane, 2006) for the United States, Russia and Mexico. However, using data from the Iranian elections in 2009, Mebane (2009, 2010) found that in cities with few invalid votes, Ahmadinejad's votes strongly diverged from Benford distribution predictions and the official candidate, in these situations, had a large vote advantage.

One important area of application of Benford's Law, which might have been somewhat neglected, is public works auditing, especially in developing countries. In the modern globalized world, developing countries need urgently to become more productive in order to have a chance to compete in the international market. This requires important investments in expensive infrastructure mega-projects, such as ports, railways, roads, telecommunications, naval industry, etc. Since developing countries are in general capital constrained, an important effort needs to be made so that public money is spent in an efficient way, bringing the cost of public infrastructure works as close as possible to competitive private sector costs, while maintaining output quality. For that goal to be attained, governments need to develop effective tools to deter data manipulation and overpricing in public procurement.

A research agenda of the authors of the present paper consists in applying Benford's Law in order to find evidence of data manipulation in public procurement worksheets (Cunha and Bugarin, 2015). The present article aims at explaining the main methodology and at illustrating its application in the analysis of the budget worksheets for the construction of one of the stadia build for the 2014 FIFA Soccer World Cup.

In accordance to Brazilian Government's proposal, FIFA selected 12 Brazilian cities to host the 2014 World Cup. The 12 cities prepared modern soccer stadiums for the event, either by building new stadia (such as São Paulo's "Itaquerão", that held the finals of the championship), by partially or totally imploding and rebuilding old stadia (such as Brasília's "Mané Garrincha") or remodeling old stadia (such as Rio de Janeiro's "Maracanã). The Amazon Arena (Arena da Amazônia) was built from 2012 to 2014, on the site of former

Vivaldo Lima Stadium, which was demolished. It is located in the city of Manaus, the capital of the state of Amazonas, in the heart of Amazon Rainforest. The stadium is strategically situated between the international airport and the city's historic center. Architect Ralf Amann from the German firm GMP Architekten[4] authors the modern architectural project, inspired by the Amazon rainforest. Sustainability features include rainwater collection for reuse in the facilities' restrooms and in watering the grass, natural ventilation to reduce energy costs, and solar energy production. It can receive up to 44,351 costumers and accommodates over 400 cars in its underground parking lot. This year, it was elected the ninth top world stadium in 2014 according to the specialized British site "Stadium DataBase"[5].

It was inaugurated on March 9, 2014 and hosted four of the 2014 World Cup games. Furthermore, it is expected to host several soccer matches of the Summer Olympic Games to be held in Rio de Janeiro in 2016. Because of its high cost of about 338 million US dollars[6] and the unlikely use of its full capacity other than in very top-level competitions[7], the construction of the Amazon Arena was heavily criticized as a "white elephant" during the street protest movement that took over the streets of Brazil in the months of June and July 2013[8]. Figure 3 presents a picture of the Arena.

Figure 3 – The Amazon Arena

Source: GMP Architekten, © Marcus Bredt.

[4] http://www.gmp-architekten.com/projects.html.

[5] http://stadiumdb.com/competitions/stadium_of_the_year_2014.

[6] According to Brazilian government official data, its cost was R$ 632,841,524.06 (http://www.portaltransparencia.gov.br/copa2014/cidades/execucoesFinanceirasDetalhe.seam;jsessionid=04F447F2E1BD0B12234EF627E923C033.portalcopa?execucaoFinanceira=13&empreendimento=5, accessed June 6, 2015). We used the February 15, 2010 Brazilian Central Bank Exchange rate.

[7] According to Downie (2013), "The local teams who will use stadiums in the Amazon city of Manaus and Cuiaba in Brazil's western farm belt rarely get more than 1,000 fans at their games".

[8] For more information on the 2013 street protests in Brazil and their relation with the World Cup expenditures, see Economist (2013).

The Amazon Arena was chosen for two main reasons. First, it has a reasonably large number of items in its database (1724 items). Second, its has been carefully audited by the Brazilian Federal Court of Accounts (TCU), which allows us to compare the findings based on Benford's Law analysis with the results of TCU auditing.

The remaining of the paper is organized as follows. Section 2 presents the tests inspired on Benford's Law that we will use in order to determine evidence of overpricing in the winning bid for the construction of the stadium. Section 3 applies these tests to the original winning worksheet. Section 4 compares the evidence suggested by our tests with the TCU's audit and section 5 suggests a new algorithm for selecting the auditing sample of a large procurement worksheet. Finally, section 6 presents our conclusions.

2. Tests of Benford's law based on the digits' frequencies

The tests used in the present study are carefully characterized in Nigrini (2012). This section presents their basic structure.

2.1. First Digit Test

According to NB Law, the expected relative frequency of a number in which the first digit is $D_1 = d_1$ is:

$$\text{Prob}(D_1 = d_1) = \log\left(1 + \frac{1}{d_1}\right) \quad \text{where} \quad d_1 \epsilon \{1,2,3,4,5,6,7,8,9\}.$$

Table 1 presents these expected relative frequencies. Furthermore, Figure 4 plots these frequencies in a two dimensional graph. For the sake of illustration, Figure 4 also plots actual first digit relative frequencies for a database composed of Japanese cities' and Brazilian municipalities' populations in 2010. One striking empirical observation is that the proportions of first-digits 1 and 5 are exactly the expected ones for the Brazilian case. Furthermore, the relative frequencies of first-digits 4 and 7 are exactly the same for both populations' datasets, and both are very close to Benford's Law expectations.

Table 1 – Expected relative frequencies of first digits according to Benford's Law

First digit	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Relative Frequency	30,10%	17,61%	12,49%	9,69%	7,92%	6,69%	5,80%	5,12%	4,58%

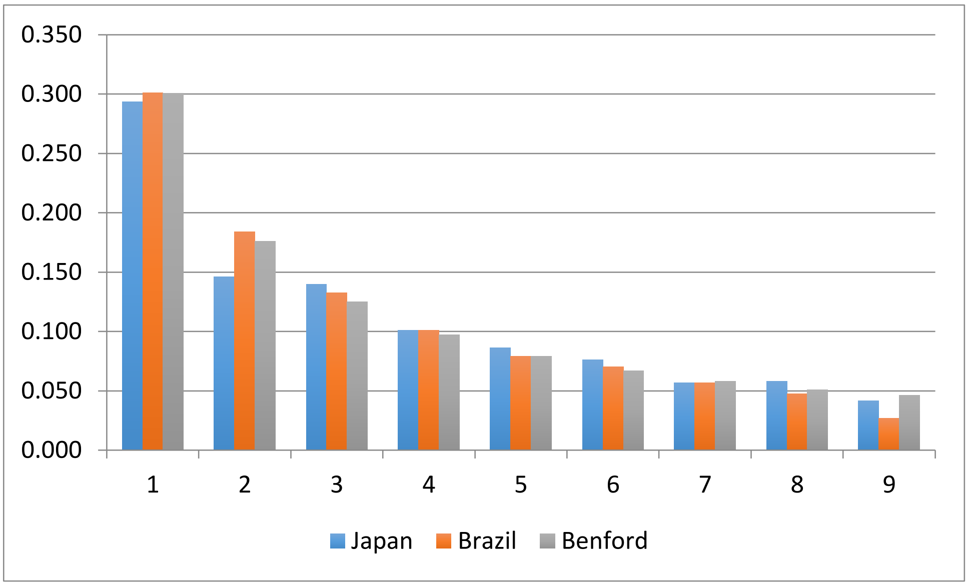
Source: Newcomb (1881)

The test consists in comparing each digit's observed relative frequency with the predicted one by means of a typical *Z*-statistic. The *Z*-statistic is calculated as shown below, where $i \epsilon \{1,2,3,4,5,6,7,8,9\}$ is the analyzed first-digit category, n is the number of observations, RF_i is the actual relative frequency of first-digit i, and ERF_i is the expected relative frequency of first-digit i.

$$z_i = \frac{|RF_i - ERF_i| - \frac{1}{2n}}{\sqrt{\frac{ERF_i(1 - ERF_i)}{n}}}$$

The 5% significance level threshold is 1.96. If the Z-statistic of a first-digit i exceeds 1.96, the frequency of the items starting with digit i does not conform to the predicted one. Therefore, that item is a candidate for further scrutiny[9].

Figure 4 – Benford's Law predicted first digit relative frequencies and actual first digit frequencies in Japanese and Brazilian 2010 cities' populations



Source: Newcomb (1881), Local Administration Bureau, Japan Ministry of Internal Affairs and Communications and Brazilian Institute of Geography and Statistics, IBGE.

Nigrini (2012) suggests two criteria for overall compliance with MB Law based on the first-digit test. Firstly, a chi-square statistic is calculated as follows, where F_i is the actual frequency of first-digit i and EF_i is its expected frequency according to Benford's Law.

$$\chi^2 = \sum_{i=1}^{9} \frac{(F_i - EF_i)^2}{EF_i} = n \sum_{i=1}^{9} \frac{(RF_i - ERF_i)^2}{ERF_i}$$

The 5% confidence threshold critical value for 8 degrees of freedom is 15.507. Therefore, if the chi-square statistic exceeds 15.507 there is evidence of an overall non-conformity of the observed distribution with NB Law[10].

Finally, a mean absolute deviation (MAD) test is based on the absolute differences between observed and expected relative frequencies, according to the following statistic.

$$MAD = \frac{1}{9} \sum_{i=1}^{9} |RF_i - ERF_i|$$

[9] For the sake of illustration, only first-digit 2's Z-statistic exceeds the 1.96 threshold in the dataset consisting of Japanese populations whereas only first-digit 9 falls outside the compliance range for the Brazilian population dataset. This first result suggests a reasonable conformity with Benford's Law.
[10] The high discrepancy for first-digits 2 for the case of Japan and 9 for the case of Brazil places both databases' chi-square statistics above the conformity threshold.

Nigrini (2012) proposes the following conformity criteria for the MAD test. If the MAD statistic is lower than 0.006, there is *close conformity*; if it is higher than 0.006 but lower than 0.0012, there is *acceptable conformity*; if it lies in the interval (0.0012, 0.0015] there is *marginally acceptable conformity* and finally, if it exceeds 0.0015 there is *nonconformity*[11].

2.2. First-Two Digits Test

According to NB Law, the expected relative frequency of a number in which the first digit, D_1, is d_1 and the second digit, D_2, is d_2 is:

$$\text{Prob}(D_1D_2 = d_1d_2) = \log\left(1+\frac{1}{d_1d_2}\right) \text{ where } d_1d_2\epsilon\{10,11,\dots,99\}.$$

Table 2 presents these expected relative frequencies. Furthermore, Figure 5 plots these frequencies in a two dimensional graph. For the sake of illustration, Figure 5 also plots actual two-digits relative frequencies for the database composed of Japanese cities and Brazilian municipalities' populations in 2010. The figure highlights the striking non-conformity of two-digit 10 for the Brazilian database. Overall, the Japanese database appears to better conform to Benford's Law than the Brazilian one.

Table 2 – Expected relative frequencies of first two-digits according to Benford's Law (in percentage)

First two digits	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
Relative Frequency	4.14	3.78	3.48	3.22	3.00	2.80	2.63	2.48	2.35	2.23
First two digits	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**
Relative Frequency	2.12	2.02	1.93	1.85	1.77	1.70	1.64	1.58	1.52	1.47
First two digits	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**
Relative Frequency	1.42	1.38	1.34	1.30	1.26	1.22	1.19	1.16	1.13	1.10
First two digits	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**
Relative Frequency	1.07	1.05	1.02	1.00	0.98	0.95	0.93	0.91	0.90	0.88
First two digits	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**
Relative Frequency	0.86	0.84	0.83	0.81	0.80	0.78	0.77	0.76	0.74	0.73
First two digits	**60**	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**
Relative Frequency	0.72	0.71	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.62
First two digits	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**
Relative Frequency	0.62	0.61	0.60	0.59	0.58	0.58	0.57	0.56	0.55	0.55
First two digits	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**
8Relative Frequency	0.54	0.53	0.53	0.52	0.51	0.51	0.50	0.50	0.49	0.49
First two digits	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**
Relative Frequency	0.48	0.47	0.47	0.46	0.46	0.45	0.45	0.45	0.44	0.44

Source: Nigrini (2012)

[11] Again, the discrepancies signaled out places the MAD-statistics for both population databases in the nonconformity range.

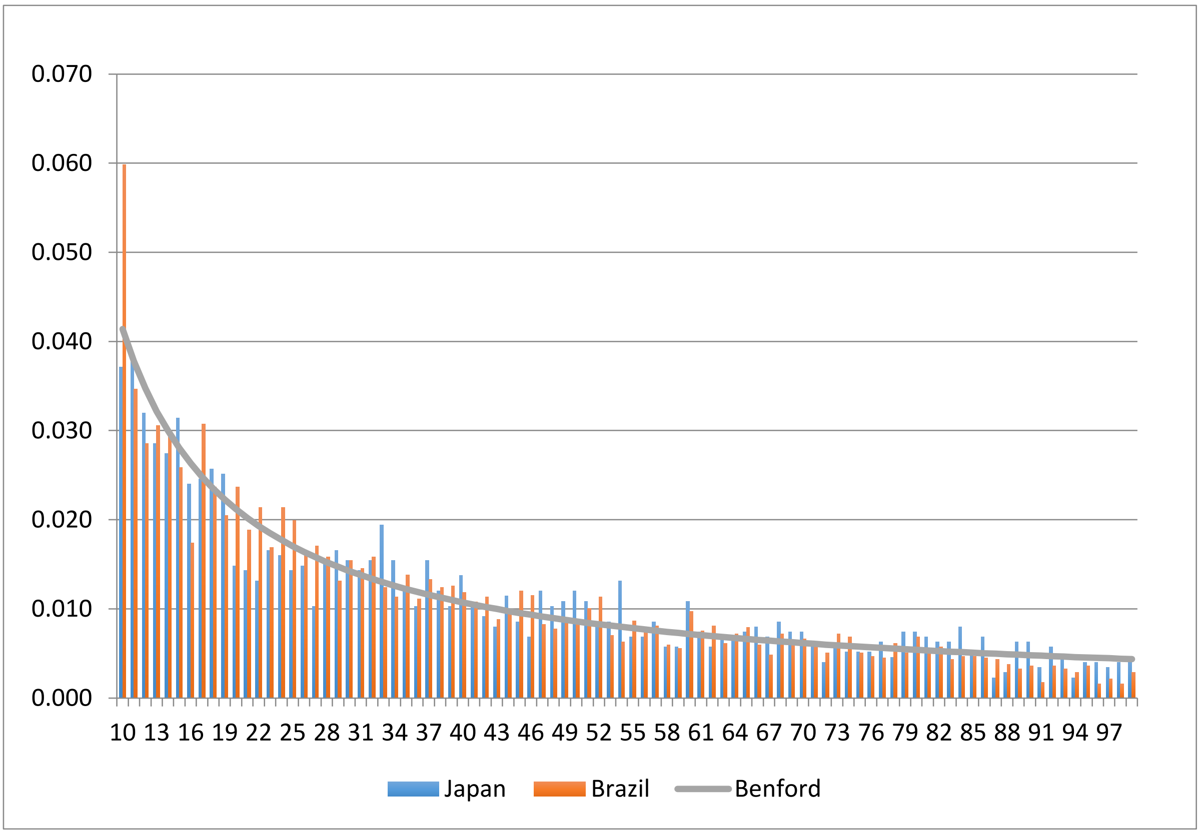
The test consists in comparing each two-digit's observed relative frequency with the (above) expected one by means of a typical Z-statistic. As in the first-digit case, the Z-statistic is calculated according to the formula below, where now $i \epsilon \{10,11,\dots,99\}$ is the analyzed two-digit category, n is the number of observations, RF_i is the observed relative frequency of two-digits i, and ERF_i is the expected relative frequency of two-digits i.

$$z_i = \frac{|RF_i - ERF_i| - \frac{1}{2n}}{\sqrt{\frac{ERF_i(1 - ERF_i)}{n}}}$$

The 5% significance level threshold is 1.96. If the Z-statistic of a two-digit exceeds 1.96, the frequencies of the items starting with these two digits do not conform to the predicted ones. Therefore, these are the candidates for further inspection.

Nigrini (2012) suggests three criteria for overall compliance with MB Law based on the two-digit tests. Firstly, if no more than 5 two-digits among all 90 classes $\{10,11,\dots,99\}$ do not conform, there is no strong evidence of manipulation. Following up with the Japanese population example, the test found only 2 two-digit categories in the non-compliance range: 33 and 54. Therefore, there is overall conformance to Benford's Law. For the Brazilian database, on the other hand, there are 11 cases of non-compliance, which suggests that the data do not conform as closely to Benford's distribution.

Figure 5 – Benford's Law predicted first two-digits relative frequencies and actual first two-digit frequencies in Japanese and Brazilian 2010 cities' populations



Source: Benford (1938), Local Administration Bureau, Japan Ministry of Internal Affairs and Communications and Brazilian Institute of Geography and Statistics, IBGE.

Secondly, a chi-square statistic is also calculated as follows, where F_i is the observed frequency of two-digits i and EF_i is the expected frequency of two-digits i.

$$\chi^2 = \sum_{i=10}^{99} \frac{(F_i - EF_i)^2}{EF_i}$$

The 5% confidence threshold critical value for 89 degrees of freedom is 112.02. Therefore, if the chi-square statistic exceeds 112.02 there is evidence of an overall non-conformity of the observed distribution with NB Law. The Japanese population chi-square statistic is 72.56, confirming compliance with Benford's Law. However, the Brazilian chi-square statistic is 204.53, which does not conform to Benford's Law.

Finally, a mean absolute deviation (MAD) test is based on the absolute differences between observed and expected relative frequencies, according to the following statistic.

$$MAD = \frac{1}{90} \sum_{i=10}^{99} |RF_i - ERF_i|$$

Nigrini (2012) proposes the following conformity criteria for the MAD test. If the MAD statistic is lower than 0.0012, there is *close conformity*; if it is higher than 0.0012 but lower than 0.0018, there is *acceptable conformity*; if it lies in the interval (0.0018, 0.0022] there is *marginally acceptable conformity* and finally, if it exceeds 0.0022 there is *nonconformity*. The corresponding figures for the Japanese and the Brazilian population databases are, respectively, 0.00178 and 0.00164, which places both databases in the range of acceptable conformity.

2.3. Summation Test

Nigrini (2012) simulated a Benford distribution and separated the resulting sample into 90 classes according to the first two digits $\{10,11,\ldots,99\}$. Then, he added all number observations in each group and found evidence that all sums led to approximately the same amounts. In other words, the numbers in each class tended to sum up to 1/90=0.011 or 1.1% of the total sum of all numbers in the sample.

However, the author found that actual data rarely conformed completely to such a standard. The usefulness of this test is precisely to point out the nonconformities. Whenever the sum of values in one category represents a too high percentage of total summation in the database, then there is room for doubting of the authenticity of the values in that category. There are no threshold explicitly suggested by Nigrini (2012); therefore, we consider here a difference higher that 100% of the expected 1.1% percentage to be the upper bound for conformity in our analysis. In other words, a realized percentage above 2.2% or, equivalently, a difference higher than 1.1%=0.011 will be considered an evidence of manipulation.

2.4. The confrontation between the two tests

Any two-digit category that falls into the nonconformity criteria range for either the *Z*-test or the summation test is a candidate for further scrutiny. However, some two-digit categories may fall into nonconformity simply because of their lack of frequency in the database. In that case, it might be an unrewarding task to dedicate time analyzing the corresponding items. Therefore, we propose to compare the frequencies of all categories that have been selected in at least one of the two tests. If one of them shows very little frequency according to both criteria, i.e., there are few observations in that category and the value of the summation of the category's items is low, then that category should be excluded from further scrutiny. We call this comparison the "confrontation" of the two tests. Our main point in doing the confrontation is that, in the case of public works' budget, the pecuniary relevance of each group should be taken into account for selecting the digits that need further auditing.

3. Analysis of Amazon Soccer arena's construction procurement

3.1 The Amazon Arena

The analysis of this study focused on the budget of Amazon soccer arena's construction originally presented to TCU by the procurement winning firm in the amount of R$ 615,992,824.67 (US$329.937.214,36 as of February 15, 2010[12]).

Subsequently, after the TCU auditing, alternative budgets that aimed at eliminating detected overpricing of most worksheet items were negotiated. We selected the initial budget for three main reasons. First, the subsequent budgets were changed after the TCU auditing; therefore, these budget sheets were not entirely formulated by the winning bidder. Since we wish to detect possible data manipulation from that bidder, the original bid should be used. Second, the first budget sheets were subject to careful TCU auditing that revealed significant overpricing. Therefore, we will be able to compare the results of our analysis based of NB Law with the results of TCU's auditing. Third, the TCU analysis is based on the ABC curve methodology, which consists of ordering the items in a budget sheet from most expensive to least expensive and selecting up to 20% of the most expensive items, until the total cost of those items adds up to about 80% of the total budget, and them compare those prices with market benchmarks. Therefore, the TCU did not make use of our proposed methodology in its analysis, which makes the comparison valuable.

The budget worksheet contains both each individual item's cost and the corresponding total cost, which consists of the quantity of an item multiplied by the unit cost of that item. For the sake of application of NB Law we could use either the unit costs or the total costs data in our database. In another application (The Maracanã Soccer Arena, see Cunha and Bugarin, 2015) we used the unit costs and our analysis was able to detect 71.54% of total overprice uncovered by the TCU auditing. Considering that Benford's Law is more likely to apply to databases which elements come from the multiplication of different random variables, such as account receivable or budget worksheets (quantities times unit prices/costs, see, for instance, Cho and Gaines, 2007), we decided to use the total costs database[13]. The database consists of 1724 items; all the corresponding total costs had at least two digits. Therefore, all data was used in our analysis.

3.2. First Digit Test

The first-digit's relative frequencies are reported in Table 3. Figure 6 presents the corresponding graph and compares is with the expected relative frequencies according to Benford's Law.

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[12] According to Brazilian Central Bank US$1.00=R$1.867 on February 15, 2010. From here on we will use that exchange rate for all calculations without further mention. We chose that date for the calculations of the dollar amounts because this is the time the TCU performed its audit on the winning bid's budget worksheets.

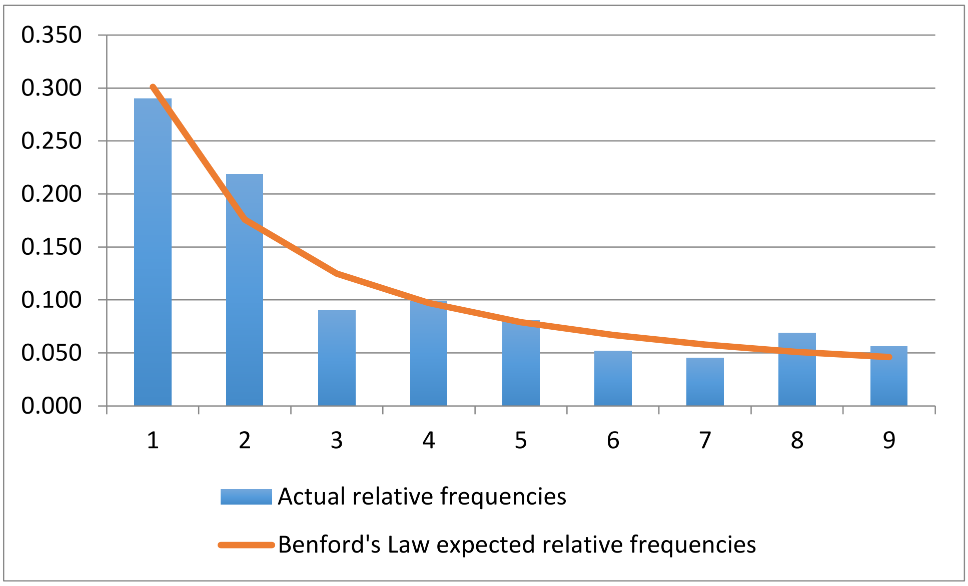
[13] We also performed the analysis based on unit costs, which yielded similar results. The details are available upon request.

Table 3 – The First Digits Relative Frequencies

First digit	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Sample relative frequency	0.313	0.208	0.103	0.083	0.076	0.058	0.053	0.054	0.052
Benford's Law relative frequency	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046

Source: Newcomb (1881) and author's calculations

Figure 6 – Benford's Law predicted first digit relative frequencies and actual first digit frequencies in Amazon Arena budget worksheet



Source: Newcomb (1881) and author's calculations

The results of the first digit tests are reported in Table 4, where: "Digit" refers to the first digit; "Frequency" is the absolute frequency (count) of items staring with the corresponding first digit in the worksheet; "Actual" is the corresponding relative frequency; "NB" is the expected relative frequency according to NB Law; "Diff" is the difference between "Actual" and "NB"; "Z-Test" refers to the *Z*-statistic; "CS" is the Chi-Square statistic intermediate calculation; and "MAD" is the Mean Absolute Deviation statistic intermediate calculation. The Chi-Square statistic is the sum of column "CS" whereas the MAD statistic is the sum of column "MAD".

The *Z*-test indicates abnormal frequencies for the digits 2 and 3, with 2 appearing too frequently whereas there is abnormally low frequency of first digit 3. This suggests most especially careful auditing of items whose total costs have first digit 2.

The chi-square statistic is the sum of all intermediate values in column CS: 25.639. The critical value for 8 degrees of freedom and 5% significance level is 15.507. Therefore, we reject the null hypothesis, suggesting non-conformity with NB Law.

Finally, the MAD test-statistic is the sum of all intermediate values in column MAD: 0.106, which highly exceeds the 0.0015 threshold adopted by Nigrini (2012), confirming the non-conformity.

Table 4 – First Digit Tests for total costs of Amazon Arena soccer stadium

Digit	Frequency	Actual	NB	Diff.	Z-Test	CS	MAD
1	540	0.313	0.301	0.012	1.080	0.856	0.012
2	359	0.208	0.176	0.032	3.483	10.179	0.032
3	178	0.103	0.125	-0.022	2.694	6.526	0.022
4	143	0.083	0.097	-0.014	1.931	3.510	0.014
5	131	0.076	0.079	-0.003	0.419	0.198	0.003
6	100	0.058	0.067	-0.009	1.446	2.082	0.009
7	91	0.053	0.058	-0.005	0.875	0.809	0.005
8	93	0.054	0.051	0.003	0.501	0.293	0.003
9	89	0.052	0.046	0.006	1.057	1.185	0.006

N=1724 observations

Source: Authors' calculations

To summarize, the tests based on the first digit suggest possible manipulation of data. Auditing based only on the first digit test, however, may be a long and fruitless task. Indeed, according to our data, over 20% of the total number of observations has first digit 2. Therefore, only those items would already fill the usual number of audited items TCU uses in its ABC curve approach. Furthermore, due to the high level of aggregation (the database is partitioned in only 9 groups), there may be additional items in other categories in which manipulations cancel out within a category. For these reasons, additional analysis is in order.

3.3. First-Two Digits Test

The results of the first-two digits tests are reported in Table 5, where, as before: "Digit" refers to the first two digits; "Frequency" is the absolute frequency of items staring with the corresponding first two-digits in the worksheet; "Actual" is the corresponding relative frequency; "NB" is the expected relative frequency according to NB Law; "Diff" is the difference between "Actual" and "NB"; "Z-Test" refers to the *Z*-statistic; "CS" is the Chi-Square statistic intermediate calculation; and "MAD" is the Mean Absolute Deviation statistic intermediate calculation.

Figure 7 plots the actual two digits relative frequencies against the ones predicted by Benford's Law.

Table 5 – First-Two Digits Tests for total costs of Amazon Arena soccer stadium

Digit	Frequency	Actual	NB	Diff.	Z-Test	CS	MAD
10	45	0.026	0.041	-0.015	3.127	9.738	0.015
11	82	0.048	0.038	0.010	2.065	4.359	0.010
12	67	0.039	0.035	0.004	0.864	0.834	0.004
13	43	0.025	0.032	-0.007	1.636	2.810	0.007
14	64	0.037	0.030	0.007	1.673	2.949	0.007
15	52	0.030	0.028	0.002	0.464	0.280	0.002
16	55	0.032	0.026	0.006	1.370	2.034	0.006
17	53	0.031	0.025	0.006	1.502	2.433	0.006
18	55	0.032	0.023	0.008	2.230	5.207	0.008
19	24	0.014	0.022	-0.008	2.269	5.403	0.008
20	37	0.021	0.021	0.000	-0.005	0.006	0.000
21	26	0.015	0.020	-0.005	1.426	2.239	0.005
22	39	0.023	0.019	0.003	0.913	0.982	0.003
23	31	0.018	0.018	-0.001	0.065	0.024	0.001
24	36	0.021	0.018	0.003	0.901	0.967	0.003

Digit	Frequency	Actual	NB	Diff.	Z-Test	CS	MAD
25	29	0.017	0.017	0.000	-0.025	0.005	0.000
26	16	0.009	0.016	-0.007	2.230	5.317	0.007
27	77	0.045	0.016	0.029	9.518	90.973	0.029
28	30	0.017	0.015	0.002	0.634	0.528	0.002
29	38	0.022	0.015	0.007	2.423	6.272	0.007
30	18	0.010	0.014	-0.004	1.230	1.748	0.004
31	30	0.017	0.014	0.004	1.183	1.632	0.004
32	12	0.007	0.013	-0.006	2.211	5.290	0.006
33	20	0.012	0.013	-0.001	0.394	0.247	0.001
34	21	0.012	0.013	0.000	0.044	0.023	0.000
35	23	0.013	0.012	0.001	0.308	0.173	0.001
36	13	0.008	0.012	-0.004	1.558	2.752	0.004
37	17	0.010	0.012	-0.002	0.555	0.441	0.002
38	12	0.007	0.011	-0.004	1.585	2.853	0.004
39	12	0.007	0.011	-0.004	1.491	2.553	0.004
40	25	0.015	0.011	0.004	1.406	2.294	0.004
41	18	0.010	0.010	0.000	-0.108	0.000	0.000
42	16	0.009	0.010	-0.001	0.268	0.149	0.001
43	12	0.007	0.010	-0.003	1.142	1.579	0.003
44	8	0.005	0.010	-0.005	2.040	4.630	0.005
45	16	0.009	0.010	0.000	-0.011	0.013	0.000
46	11	0.006	0.009	-0.003	1.152	1.617	0.003
47	20	0.012	0.009	0.002	0.946	1.139	0.002
48	7	0.004	0.009	-0.005	2.029	4.612	0.005
49	10	0.006	0.009	-0.003	1.195	1.737	0.003
50	18	0.010	0.009	0.002	0.697	0.679	0.002
51	10	0.006	0.008	-0.003	1.064	1.417	0.003
52	15	0.009	0.008	0.000	0.063	0.038	0.000
53	12	0.007	0.008	-0.001	0.401	0.284	0.001
54	11	0.006	0.008	-0.002	0.606	0.546	0.002
55	6	0.003	0.008	-0.004	1.911	4.159	0.004
56	15	0.009	0.008	0.001	0.344	0.231	0.001
57	16	0.009	0.008	0.002	0.689	0.681	0.002
58	17	0.010	0.007	0.002	1.038	1.379	0.002
59	11	0.006	0.007	-0.001	0.307	0.199	0.001
60	5	0.003	0.007	-0.004	1.962	4.396	0.004
61	9	0.005	0.007	-0.002	0.769	0.828	0.002
62	9	0.005	0.007	-0.002	0.719	0.741	0.002
63	8	0.005	0.007	-0.002	0.962	1.219	0.002
64	7	0.004	0.007	-0.003	1.210	1.829	0.003
65	9	0.005	0.007	-0.001	0.573	0.517	0.001
66	5	0.003	0.007	-0.004	1.722	3.480	0.004
67	10	0.006	0.006	-0.001	0.178	0.108	0.001
68	28	0.016	0.006	0.010	5.028	26.657	0.010
69	10	0.006	0.006	0.000	0.083	0.055	0.000
70	3	0.002	0.006	-0.004	2.192	5.468	0.004
71	6	0.003	0.006	-0.003	1.231	1.910	0.003
72	7	0.004	0.006	-0.002	0.882	1.072	0.002
73	14	0.008	0.006	0.002	1.041	1.427	0.002
74	9	0.005	0.006	-0.001	0.174	0.110	0.001
75	10	0.006	0.006	0.000	-0.133	0.001	0.000

Digit	Frequency	Actual	NB	Diff.	Z-Test	CS	MAD
76	8	0.005	0.006	-0.001	0.413	0.326	0.001
77	16	0.009	0.006	0.004	1.884	4.159	0.004
78	11	0.006	0.006	0.001	0.312	0.224	0.001
79	7	0.004	0.005	-0.001	0.627	0.621	0.001
80	6	0.003	0.005	-0.002	0.921	1.172	0.002
81	4	0.002	0.005	-0.003	1.550	2.929	0.003
82	9	0.005	0.005	0.000	-0.141	0.001	0.000
83	5	0.003	0.005	-0.002	1.161	1.755	0.002
84	13	0.008	0.005	0.002	1.226	1.934	0.002
85	6	0.003	0.005	-0.002	0.765	0.868	0.002
86	6	0.003	0.005	-0.002	0.735	0.815	0.002
87	9	0.005	0.005	0.000	-0.020	0.023	0.000
88	20	0.012	0.005	0.007	3.805	15.740	0.007
89	15	0.009	0.005	0.004	2.126	5.261	0.004
90	14	0.008	0.005	0.003	1.822	3.964	0.003
91	6	0.003	0.005	-0.001	0.590	0.582	0.001
92	9	0.005	0.005	0.001	0.143	0.101	0.001
93	10	0.006	0.005	0.001	0.529	0.496	0.001
94	13	0.008	0.005	0.003	1.630	3.253	0.003
95	9	0.005	0.005	0.001	0.236	0.172	0.001
96	15	0.009	0.005	0.004	2.426	6.758	0.004
97	4	0.002	0.004	-0.002	1.150	1.763	0.002
98	5	0.003	0.004	-0.002	0.764	0.890	0.002
99	4	0.002	0.004	-0.002	1.105	1.651	0.002

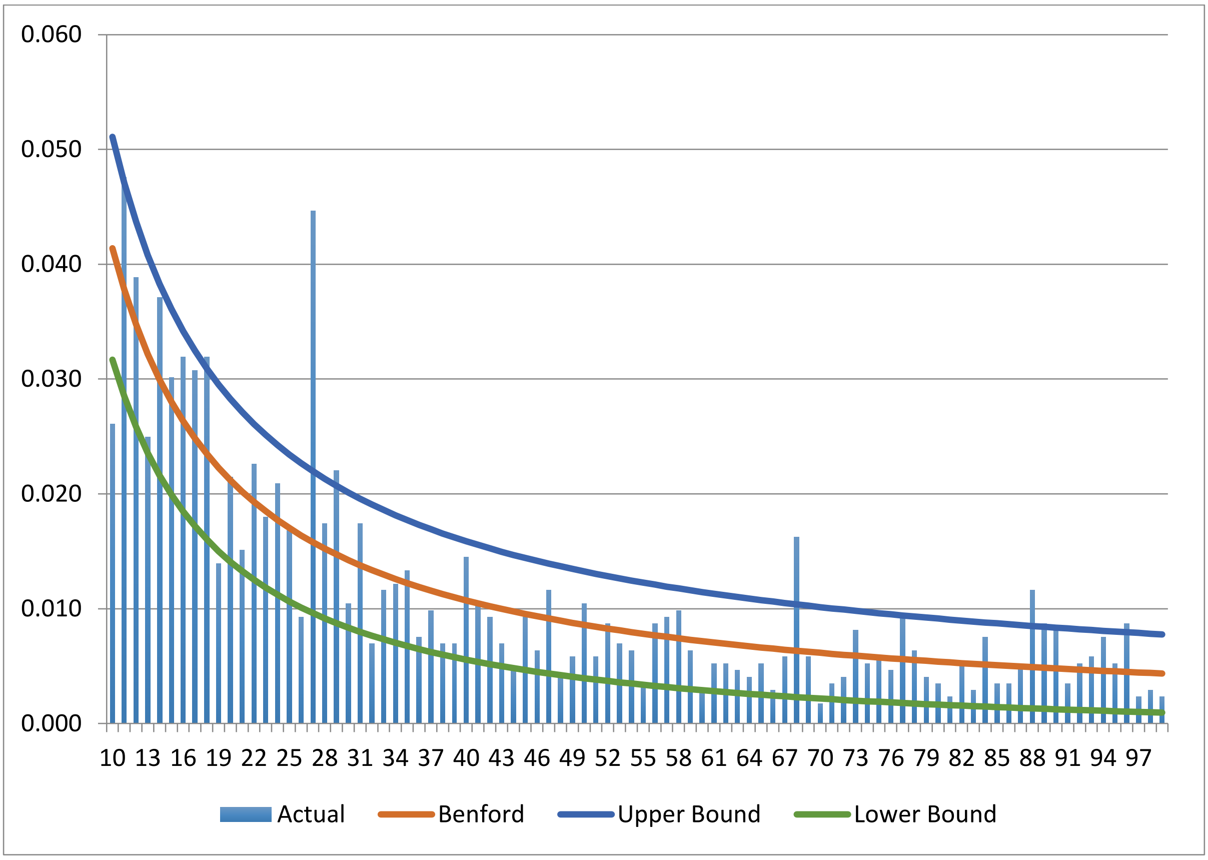
Number of observations 1724
Source: author's calculations

According to Table 5, there is evidence of non-conformity in the digits 10, 11, 18, 19, 26, 27, 29, 32, 44, 48, 60, 68, 70, 88, 89 and 96 with respect to the proportions of the descending curve of NB Law. These corresponds to 16 two-digit categories exceeding the limit of 1.96, a number very much above the threshold of 5 peaks suggested by Nigrini (2012). Therefore, the *Z*-test suggests that the data have been manipulated. It is noteworthy that some of the peaks correspond to numbers that appear too frequently whereas others correspond to numbers appear too seldom. Naturally, we would expect that the ones that appear too frequently are the top candidates for being manipulated data. In particular, one should stress first-two digits 27 and 68 (see Figure 7).

The Chi-Square statistic, the summation of column CS, is 293.736. The critical value for 89 degrees of freedom and 5% significance level is 112.02. Therefore, we reject the null hypothesis, suggesting, again, con-compliance with NB Law.

The last test applied is MAD. The test statistic found for Amazon Arena is 0.299, which highly exceeds the 0.0022 threshold adopted by Nigrini (2012). This result suggests, once again, possible manipulation of data.

Figure 7 – Benford's Law predicted first-two digit relative frequencies and actual first-two digit frequencies in Amazon Arena budget worksheet



Number of observations: 1724, the Upper bound and Lower bound curves refer to the 95% significance level
Source: Authors calculations

3.4. Summation Test

In order to assess the pecuniary significance of each pair of digits in the budget worksheet we perform the complementary Summation Test. The results are shown in Table 6 below, where the 1st column refers to the first-two digits of the observations; the 2nd column corresponds to the sum of the total costs of items that have the first-two digits indicated in the 1st column; the 3rd column shows the proportions of the sums calculated in the 2nd column with respect to the total costs of the worksheet; and column 4 computes the difference between the actual proportions of the sums and the expected ones.

Recall that the expected proportions of each sum of total cost in each two-digit category is 1.1% or 0.011, according to Nigrini (2012). Recall, furthermore, that we have set an upper bound threshold of 0.0022 for conformity. Therefore, Table 6 highlights peaks in the first two digits 11, 12, 13, 14, 15, 52, 60, 67, 77 and 78. It is noteworthy the very high ratio of appearance of two-digits 60, representing 20.7% of total costs. The test strongly suggests nonconformity to NB Law.

Table 6 - Summation Test for total costs of Amazon Arena soccer stadium

Digit	Sum	Actual	Benford	Difference
10	6,779,337.27	0.011	0.011	0.000
11	13,321,384.80	0.022	0.011	0.011
12	16,925,818.66	0.027	0.011	0.016
13	19,373,384.39	0.031	0.011	0.020

Digit	Sum	Actual	Benford	Difference
14	22,580,106.79	0.037	0.011	0.026
15	28,505,937.80	0.046	0.011	0.035
16	8,798,956.04	0.014	0.011	0.003
17	5,324,811.18	0.009	0.011	-0.002
18	5,174,192.59	0.008	0.011	-0.003
19	9,160,737.42	0.015	0.011	0.004
20	5,700,596.87	0.009	0.011	-0.002
21	5,706,353.64	0.009	0.011	-0.002
22	4,128,765.13	0.007	0.011	-0.004
23	7,594,421.93	0.012	0.011	0.001
24	10,850,773.13	0.018	0.011	0.007
25	6,773,131.14	0.011	0.011	0.000
26	3,718,898.40	0.006	0.011	-0.005
27	9,754,202.70	0.016	0.011	0.005
28	1,467,372.18	0.002	0.011	-0.009
29	5,378,772.64	0.009	0.011	-0.002
30	6,774,856.16	0.011	0.011	0.000
31	5,134,543.48	0.008	0.011	-0.003
32	1,114,827.30	0.002	0.011	-0.009
33	581,314.36	0.001	0.011	-0.010
34	5,699,104.65	0.009	0.011	-0.002
35	5,754,647.32	0.009	0.011	-0.002
36	1,568,419.89	0.003	0.011	-0.008
37	1,370,164.55	0.002	0.011	-0.009
38	220,564.72	0.000	0.011	-0.011
39	756,380.76	0.001	0.011	-0.010
40	9,925,971.95	0.016	0.011	0.005
41	10,243,159.92	0.017	0.011	0.006
42	1,170,449.44	0.002	0.011	-0.009
43	1,530,502.13	0.002	0.011	-0.009
44	1,071,595.82	0.002	0.011	-0.009
45	601,959.44	0.001	0.011	-0.010
46	1,506,748.31	0.002	0.011	-0.009
47	1,714,607.67	0.003	0.011	-0.008
48	6,377,647.71	0.010	0.011	-0.001
49	227,670.25	0.000	0.011	-0.011
50	1,226,073.95	0.002	0.011	-0.009
51	698,252.58	0.001	0.011	-0.010
52	53,463,119.57	0.087	0.011	0.076
53	7,026,444.91	0.011	0.011	0.000
54	355,504.46	0.001	0.011	-0.010
55	734,563.64	0.001	0.011	-0.010
56	942,726.17	0.002	0.011	-0.009
57	1,990,461.73	0.003	0.011	-0.008
58	2,000,276.50	0.003	0.011	-0.008
59	7,540,247.05	0.012	0.011	0.001
60	127,223,393.04	0.207	0.011	0.196
61	332,069.66	0.001	0.011	-0.010
62	948,407.85	0.002	0.011	-0.009
63	783,863.39	0.001	0.011	-0.010
64	329,013.41	0.001	0.011	-0.010

Digit	Sum	Actual	Benford	Difference
65	2,122,060.55	0.003	0.011	-0.008
66	332,589.49	0.001	0.011	-0.010
67	9,486,088.97	0.015	0.011	0.004
68	315,803.31	0.001	0.011	-0.010
69	854,916.40	0.001	0.011	-0.010
70	78,632.23	0.000	0.011	-0.011
71	8,697,745.38	0.014	0.011	0.003
72	313,056.63	0.001	0.011	-0.010
73	2,611,687.45	0.004	0.011	-0.007
74	7,655,137.69	0.012	0.011	0.001
75	279,397.79	0.000	0.011	-0.011
76	329,847.91	0.001	0.011	-0.010
77	78,725,633.04	0.128	0.011	0.117
78	17,555,153.59	0.028	0.011	0.017
79	414,123.69	0.001	0.011	-0.010
80	185,950.12	0.000	0.011	-0.011
81	913,013.16	0.001	0.011	-0.010
82	133,992.08	0.000	0.011	-0.011
83	343,503.97	0.001	0.011	-0.010
84	2,848,420.58	0.005	0.011	-0.006
85	351,228.64	0.001	0.011	-0.010
86	185,162.72	0.000	0.011	-0.011
87	2,041,493.74	0.003	0.011	-0.008
88	407,834.26	0.001	0.011	-0.010
89	2,018,837.03	0.003	0.011	-0.008
90	1,255,957.83	0.002	0.011	-0.009
91	2,030,709.91	0.003	0.011	-0.008
92	999,639.96	0.002	0.011	-0.009
93	2,107,473.07	0.003	0.011	-0.008
94	1,354,079.96	0.002	0.011	-0.009
95	1,095,960.57	0.002	0.011	-0.009
96	1,613,890.82	0.003	0.011	-0.008
97	39,068.09	0.000	0.011	-0.011
98	306,419.34	0.000	0.011	-0.011
99	30,806.34	0.000	0.011	-0.011

Number of observations: 1724
Source: Authors calculations

3.5. Confrontation between the First-Two Digits Test and the Summation Test

Next, we select the digits detected as critical in the First-Two Digits Test and Summation Test. Then we carry out a confrontation between these tests to confirm the sample relevance of the selected digits, comparing their relative frequency in each one of the tests. All two digits that show low relative frequency in both tests correspond to items that do not appear frequently in the database and, furthermore, to item which aggregate costs are not very significant as percentage of total budget. Therefore, these two-digit items are considered non-critical points: it is not worthy to spend the auditors' time analyzing these items.

Table 7 shows the digits that were selected by either one of the tests in column 1. Column 2 shows the relative frequencies of these digits according to the first-two digit tests. Column 3 displays the proportions of the sum of total costs of items starting with these digits according to the Summation test. Column 4 singles out the two-digits that have little significance in the spreadsheet according to both criteria: low percentage of items starting with those two digits and low sum of the corresponding values as percentage of the sum of all items (No).

The confrontation between the tests suggests excluding digits 10, 19, 26, 32, 44, 48, 68, 70 and 88 from our analysis. Therefore, our methodology suggests the following critical points for the auditing process: 11, 12, 13, 14, 15, 18, 27, 29, 52, 60, 77, 78, 88, 89 and 96.

Table 7 – Confrontation between the First-Two Digits Test and Summation Test

Digits	First-Two Digits Test	Benford	Summation Test	Critical Digits
10	0.026	0.041	0.011	No
11	0.048	0.038	0.022	Yes
12	0.039	0.035	0.027	Yes
13	0.025	0.032	0.031	Yes
14	0.037	0.030	0.037	Yes
15	0.030	0.028	0.046	Yes
18	0.032	0.023	0.008	Yes
19	0.014	0.022	0.015	No
26	0.009	0.016	0.006	No
27	0.045	0.016	0.016	Yes
29	0.022	0.015	0.009	Yes
32	0.007	0.013	0.002	No
44	0.005	0.010	0.002	No
48	0.004	0.009	0.010	No
52	0.009	0.008	0.087	Yes
60	0.003	0.007	0.207	Yes
68	0.016	0.006	0.001	Yes
70	0.002	0.006	0.000	No
77	0.009	0.006	0.128	Yes
78	0.006	0.006	0.028	Yes
88	0.012	0.005	0.001	Yes
89	0.009	0.005	0.003	Yes
96	0.009	0.005	0.003	Yes

Source: author's calculations

4. Comparison with the Brazilian Court of Accounts' analysis

Brazilian TCU performed a careful analysis of the initial winning bidder's budget sheet, requiring a series of price and quantity adjustments in order to approve the contract. We will compare TCU findings after the analysis of that initial winning bidder's budget sheet with our analysis based on Benford's Law.

Table A1 in the Appendix presents the output of TCU's audit. This audit compared the winning bid's budget sheets with market prices as of February 2010. Therefore, all figures presented here in dollars use the exchange rate of February 15, 2010 according to the Brazilian Central Bank.

To perform that comparison a few details on the TCU methodology is in order. As we explained earlier, the TCU uses the ABC curve methodology, which consists of ordering the items according to their total costs in decreasing order, and audit up to 20% of all such ordered items, starting from the most expensive to the least expensive ones, until the total cost of those items add up to 80% of the total budget. In the present case the TCU analysis total audited items' costs amounted to R$492,594,332.98 (US$263,842,687) out of a total budget of R$615,992,824.67 (US$329,937,214.36), which corresponds to 79.97% of the entire budget.

The TCU introduced two types of aggregation when ordering the items.

(i) First, due to the complexity of the project, some items were repeated several times in the worksheet. Each time the item referred to a different part/stage of the construction. The unit price was always the same, naturally, but the quantities changes according to the expected use. Therefore, the same item appeared with different codes and total prices throughout the budget worksheet. For example, the item "Ferragens de aço CA-50A" (Steel hardware of type CA-50A) appeared under 19 different item codes, corresponding to total costs in in 15 two-digit categories[14]. The TCU's analysis output aggregates all observations of the same item in the budget sheet and analyses it as one item.

(ii) Second, the TCU performs two types of analysis for each item: price and quantity. In other words, the TCU analyses if the total quantity proposed in the worksheet is adequate and also if the unit price reflects market prices.

(iii) Third, in some cases the TCU found actually that the total cost of an item has been under-calculated by the bidder, usually because the market prices determined by TCU are higher than what appeared in the worksheet. In that case, a negative number appears in the TCU overprice estimation. Therefore, TCU's overprice estimate corresponds to the net amount resulting of the subtracting total underprices from the total overprice.

(iv) Fourth, the TCU found it difficult to analyze the cost of air conditioning services. Therefore, it aggregated all items related to air conditioning, calculated their total cost in the budget worksheet, then calculated the total amount of refrigeration to be generated by the system (in tons of refrigeration, TR) and then divided the total cost by the total amount of refrigeration (652 TR) in order to obtain the per-unit-of-refrigeration cost. This was used to calculate overprice in that category, which amounted to R$2,613,808.35 (US$1,400,004.37).

Given the above explanations, we performed our comparison as follows:

(i) Each observation of a repeated item with overprice detected by TCU placed that item in a (possibly different) two-digit category. If any one of these categories was signaled out by our Benford's Law tests, then we say that the item has been uncovered by our methodology, and compute the total overprice determined by TCU.

(ii) What matters to our analysis is the manipulation of worksheet total values, be it by manipulating quantities or prices. Therefore, since we are analyzing total costs, it is irrelevant to us where the manipulation came from.

(iii) Since we aim at uncovering overprices, we did not subtract underprice uncovered by TCU. Therefore, for our analysis, the total overprice uncovered by TCU was used, corresponding to a total amount of R$90,394,830.23 (US$48,417,152.41). This is higher than the amount that appears in TCU's audit output (Table A1, Appendix), which is R$86.544.009,11.

(iv) Finally, there is no way our analysis can incorporate the aggregation of all different items that compose the air-conditioning system. Therefore, we will not be able to uncover the

[14] The two-digit categories are: 10, 11, 12, 13, 16, 19, 20, 23, 24 (three different codes), 25, 34, 42, 59, 65, 67, 78 and 94.

corresponding overprice (R$2,613,808.35) detected by TCU. If one subtract that amount from total overprice (R$90,394,830.23) then we obtain T=R$87,781,021.88 (US$47,017,148.04). T is the upper bound for whatever the methodology we propose, based on Benford's Law, can possibly uncover of overprices. We use T as the reference for our comparison.

Table 8 presents the comparison with the TCU analysis.

Table 8 – Confrontation between the results of tests of the NB Law and TCU's overpricing analysis for the digits 11, 12, 13, 14, 15, 18, 27, 29, 60, 77, 78 and 89, in Brazilian Reals as of February 2010

Digit	Item code	Overpricing detected by TCU (in Brazilian Real, R$)
11	11.4.1	6,235,225.71
	15.105	13,555.84
	24.10	202,334.75
	24.26	78,798.00
	24.34	5,859,425.46
12	8.17	338,948.36
	8.11 (Already uncovered)	6,235,225.71
	24.31 (Already uncovered)	5,859,425.46
13	8.12	6,249,520.46
	11.4.2 (Already uncovered)	5,859,425.46
	24.27 (Already uncovered)	6,235,225.71
14	13.15	124,310.17
	13.19	706,545.34
	24.15	1,993,872.00
15	7.1	610,038.38
	10.5	1,253,793.13
	13.2	504,835.29
	15.108	758,675.38
	11.7.2 (Already uncovered)	5,859,425.46
18	15.51	431,398.36
27	11.8.5	6,387,800.00
	14.1	1,141,339.79

Digit	Item code	Overpricing detected by TCU (in Brazilian Real, R$)
	11.6.2(Already uncovered)	5,859,425.46
29	8.6	145,951.68
	24.5 (Already uncovered)	202,334.75
	24.12 (Already uncovered)	78,798.00
52	12.3	8,827,023.45
60	4.2	22,180,663.27
77	24.21 (Already uncovered)	78,798.00
78	6.16	5,915,199.94
	9.10 (Already uncovered)	1,993,872.00
	11.2.1 (Already uncovered)	6,235,225.71
89	24.1	443,855.34
Total		70,403,110.10

Source: Brazilian Court of Accounts, TCU and authors' calculations

TCU's ABC Curve analysis identified overpricing in several items that had one of these critical digits as the first two digits of the total costs; the total overpricing for these services was R$ 70,403,110.10 (US$37,709,215.27) as of February 2010. This corresponds to 80.20 % of total overpricing (R$87,781,021.88 or US$47,017,148.04) uncovered by TCU.

It is very important to stress that TCU auditors work on a very tight time schedule. Therefore, the better the selection of data to be analyzed, the better the result of their analysis. The ABC curve is a rather efficient and standard methodology based on the selecting the most expensive items. The methodology we propose, based on Benford's Law, suggests a different ordering of data. In the present application, if all the two-digit categories that our methodology suggests were audited, this would amount to only R$402,021,662, which corresponds to only 65.26% of the total budget. In spite of the reduced budget auditing sample, our methodology would have allowed us to single out 80.20% of total overpricing found by TCU.

It is noteworthy that TCU did note audit all items in the categories we suggest. We might only wonder if additional overprices would not have found if a complete audit of all two-digits highlighted by the methodology based on Benford's Law had been audited. Note, moreover, that had we chose a lighter criterion for the summation test, more two-digit would have been singled out, increasing the performance of the auditing process. This suggests the algorithmic approach we present in next section.

5. Proposition: An algorithm for selecting the auditing sample

Based on the previous considerations, we propose now an algorithm for determining the sample to be audited. There are five main parameters.

The parameter σ reflects the percentage of the total budget cost to be audited. It is set here at 80% in order to preserve the present standard used in the ABC-curve methodology, as explained earlier; however, other standards could also be used.

The parameter λ reflects the significance level to be used in the two-digit test. Following Nigrini (2012), it is initially set at 5%.

The parameter μ reflects the initial threshold to be used in the summation test. Following our proposal, it is initially set at 100%.

The parameter δ reflects the adjustment to be made to the summation test significance level parameter μ, as will become clear in what follows. It is set here at 25%.

Finally, the parameter ε reflects the precision of the stop criteria, i.e., how close the total cost of the selected sample is from the targeted percentage σ if the total budget. We set this parameter at 5%.

The main goal of the algorithm is to select a sample that contains the observations that are most likely to have been manipulated according to the tests inspired in Benford's Law, while auditing about σ (percent) of the total cost of the budget worksheet, whenever possible. Figure 8 details the algorithm.

Figure 8 – An algorithm based on Benford's Law to select the audit sample

Step	Action
Step 1:	Setting up the initial values.
	Set $\sigma = 80\%$, $\lambda = 5\%$, $\mu = 100\%$, $\delta = 25\%$, $\varepsilon = 5\%$. Set $T =$ the total cost of the budget. Set Below=false.
Step 2:	Two-digit test.
	Apply the two-digit test using the significance criterion λ. Select the corresponding two digit categories.
Step 3:	Summation test.
	If $\lambda > 10\%$ then set Below=true and go to Step 7. If $\mu \leq 0$ then set $\lambda = \lambda + 5\%$, set $\mu = \mu + \delta$ and go to Step 2. Apply the summation test with the μ threshold, i.e., select all two-digit categories with relative frequency above $0.011(1 + \mu)$ in the summation calculation.
Step 4:	Confrontation between the two-digit and the summation tests
	Perform the confrontation between the two-digit test and the summation test to select the auditing sample, A.
Step 5:	Auditing budget cost
	Calculate the total cost of the sample in the budget worksheet, S.
Step 6:	Compare the sample and the entire worksheet costs.
	Compute $p = \frac{S-T}{T}$ If $\lvert p \rvert \leq \varepsilon$ then go to Step 7. If $\lvert p \rvert > \varepsilon$ and $S < T$ then set $\mu = \mu - \delta$ and go to Step 3. If $\lvert p \rvert > \varepsilon$ and $S > T$ then set $\mu = \mu + \delta$ and go to Step 3.
Step 7:	Audit sample S.

	If Below= false, then we were able to select a sample with total cost near the target of σB. If Below=true, the methodology based on Benford's Law was unable to signal out a number of two-digit categories high enough so that the corresponding cost nears the target of σB.

Source: Authors' proposal

The main idea behind the algorithm is to start with the basic parameters and select the first audit sample. If that audit sample's total cost is already near the target 80% of the budget's total cost, then that is the final sample. If it's total cost is too large, then straighten the selection criterion of the summation test to reduce the sample size. If the cost is too low, then relax the selection criterion of the summation test to augment the sample size. If it is not possible to augment the sample size by relaxing the summation test criterion anymore, but the sample total cost is still too low, then relax the confidence level of the two-digit test to 10%. No more relaxations are allowed. If the final sample's total cost is still too low, then the algorithm was not able to select enough items based exclusively on Benford's Law criteria. In that case, the sample may be completed with other criteria, such as the cost of items, as in the ABC curve methodology. This final completion is not introduced here.

For the sake of illustration, let us apply the algorithm to the Amazon Arena worksheet.

The first iteration allows us to select the two-digit categories are: 11, 12, 13, 14, 15, 18, 27, 29, 52, 60, 77, 78, 88, 89 and 96. The corresponding sample total cost is: $S =$R$402,021,662 and it corresponds to 65.26% of total costs. This is the analysis we already performed in section 4.

The second iteration reduces the upper bound threshold for the summation test from 0.022 to 0.0195, but no new two-digit category is added.

The third iteration reduces the upper bound threshold for the summation test to 0. 0165 and allows us to add the two-digit categories 24 and 41. The corresponding sample total cost is: $S =$R$423,115,595 and it corresponds to 68.69% of total costs.

The fourth iteration reduces the upper bound threshold for the summation test to 0. 01375 and allows us to add the two-digit categories 16, 19, 40 and 67. The corresponding sample total cost is: $S =$R$460,487,349 and it corresponds to 74.76% of total costs.

The fifth iteration suggests reducing the two-digit test significance level to 10%, rather than 5%. Although three new categories are selected according to the two-digit test (55, 66 and 90), none of these categories pass the confrontation with the summation test and the algorithm concludes with Below=true, i.e., we were not able to detect additional categories to audit based on Benford's Law.

The final auditing sample suggested by this methodology is $S = \{11,12,13,14,15,16,18,19,24,27,29,40,41,52,60,67,77,78,88,89,96\}$. The total cost of items in that sample is $S =$R$460,487,349 and it corresponds to 74.76% of total costs. Note that many of the items in sample S where not audited by TCU. Therefore, it is not possible to assert whether that sample, were it indeed audited, would lead to findings of overprice not detected by the traditional ABC curve. However, we can observe that our methodology was able to uncover 81.65% of total overprice found in the TCU audit.

6. Conclusion

The present research tested the application of Newcomb-Benford's Law to the total costs of the budget worksheet for the construction of Amazon Arena soccer stadium. The main goal of the methodology is to point to which items might be more likely to have been manipulated, thereby might correspond to overestimated costs. It applied the First Digit Tests, First-Two Digits Test and the Summation Test, all based of Benford's Law, using the *Z*-statistic, the Chi-Square and the Mean Absolute Deviation tests.

All tests point to a non-conformity of the database to Benford's Law, which suggests that the costs presented in the budget worksheets may have been manipulated. Next, our analysis singled out thirteen first-two-digit categories of total prices 11, 12, 13, 14, 15, 18, 27, 29, 52, 60, 77, 78 and 89 which contained items that were shown by TCU analysis to have been overpriced by a total amount of R$ 70,403,110.10 (US$37,709,215.27) as of February 2010, which corresponds to over 80% of total overprice uncovered by TCU. In particular, that analysis was able to uncover the single item with highest overprice, item 4.2, in the two-digit category 60, with overprice of R$22,180,663.27 (US$11,880,375.80). Comparing the total cost associated to the actually audited sample with the sample suggested by our methodology, we find out that it corresponds to about 65% of the entire procurement cost, whereas the TCU audited about 80% of it, as established by the ABC curve's methodology.

To conclude, our paper proposes an algorithm to select the sample to be audited in an alternative, possibly more efficient fashion, while still auditing about 80% of total procurement cost. To be certain, more empirical work is needed in order to fully compare the two methodologies. For example, two teams could perform simultaneously the auditing, one using the Benford's Law derived methodology and a second one using the traditional ABC curve and the results could be compared ex-post. Naturally, for the sake of the *res publica*, the two teams would have to aggregate their findings once the auditing would be concluded. This additional robustness test is left here as a suggestion for future research.

It is noteworthy to discuss the possible future of data manipulation in government procurements. Indeed, at least in the case of Brazil, Newcomb-Benford's Law has not yet being applied as a regular, complementary tool by the Brazilian Court of Accounts, which, in part, may explain the highly significant result found in the present analysis. However, if this tool becomes standard, bidding firms might become aware of it and may devise more sophisticated ways to overprice their bids in order to try avoiding detection. However, as we tried to illustrate by the complementary use of the first digit, two-digits and the summation tests, Benford's Law is a very rich tool and several diverse tests may be applied to a single database. For example, there is also a simple test for the *second* digit. Whereas it may be a feasible task to manipulate data while keeping the expected relative frequencies for the first digits, it may be a much harder task to keep data in conformity to the expected relative frequencies of the first two digits, the second digit, and the summation. Therefore, the authors do not anticipate simple manipulation algorithms in the near future. Furthermore, additional tests are been developed everyday, and new laws can alternatively be used, such as the Zipf's Law (Adueke & Weir, 2012), which is, in a specific way, a generalization of Benford's Law.

Recent trends towards increased globalization requires developing countries' government to invest heavily in expensive large-scale infrastructure projects in order to keep on the map of an ever more competitive world. In a context of capital constraint, it is essential to keep public procurement works at their lowest possible cost while ensuring the required quality of the final output. Benford's Law has been successfully used to test data manipulation in very diverse areas, from accounting and financial data to tax returns, macroeconomic and electoral data. The present work illustrates the successful use of the Law as a tool to help auditors of large public work projects and suggest that Benford Law should be regularly used in the public auditor's toolbox.

References

Benford, F. (1938) "The law of anomalous numbers". *Proceedings of the American Philosophical Society* **78** (4), 551-572.

Berton, L. (1995) "He's got their number: scholar uses math to foil financial fraud". *Wall Street Journal.* 10, B1, July 10.

Cho, W. T. and B. Gaines (2007) "Breaking the (Benford) Law: Statistical Fraud Detection in Campaign Finance". *The American Statistician* **61**(3): 218-223.

Cunha, F. R. da and M. Bugarin (2015) "Benford's Law for audit of public works: an analysis of overpricing in Maracanã soccer arena's renovation". *Economics Bulletin* **35** (2): 1168-1176.

Downie, A (2013) "Brazil protests pose challenge for World Cup organisers". *Reuters*, June 19, 2013. Available at <http://in.reuters.com/article/2013/06/18/brazil-protests-soccer-idINDEE95H0FW20130618>. Accessed June 6, 2015.

Durstchi, C.; W. Hillison and C. Pacini (2004) "The effective use of Benford's Law to assist in detecting fraud in accounting data". *Journal of Forensic Accounting* **V**: 17-34.

Economist (2013) "Protests in Brazil – The streets erupt". *The Economist*, June 18, 2013. Available at < http://www.economist.com/blogs/americasview/2013/06/protests-brazil>. Accessed June 6, 2015.

European Commission (2010) "Report on Greek Government Deficit and Debt Statistics". Available at <http://epp.eurostat.ec.europa.eu/portal/page/portal/product_details/publication?p_product_code=COM_2010_report_greek>. Accessed on July 10, 2014.

Hill, T.P. (1995) "Base-Invariance Implies Benford's Law". *Proc. Amer. Math. Soc.* **123**, 887-895.

Mebane, W. R. (2006) "Election Forensics: vote counts and Benford's Law". Summer Meeting of the Political Methodology Society. Papers, Posters and Syllabi. Nº 620.

______ (2010) "Fraud in the 2009 presidential election in Iran?" *Chance* **23**(1), 6-15.

______ (2009) "Note on the presidential election In Iran". Updated notes on author's website. Available at http://www.benfordonline.net/fullreference/756. Accessed July 10, 2014.

Newcomb, S. (1881) "Note on the frequency of the different digits in natural numbers". *The American Journal of Mathematics* **4**(1), 39-40.

Nigrini, M. J. (1992) *The Detection of Income Tax Evasion Through an Analysis of Digital Frequencies*. Ph.D. thesis. Cincinnati, OH: University of Cincinnati.

______ (2000) *Digital analysis using Benford's Law: Tests Statistics for Auditors*. Global Audit Publications: Vancouver.

______ (2012) *Benford's Law. Applications for Forensic Accounting Auditing, and Fraud Detection*. John Wiley & Sons, Inc.: Hoboken, New Jersey.

Odueke, A, and G. Weir (2012) "Triage in Forensic Accounting using Zipf's Law" in: Weir, G. and A. Al-Nemrat, *Issues in Cybercrime, Security and Digtal Forensics.* University of Strathclyde Publishing, 33-43.

Rauch, B.; G. Brähler and M. Göttsche (2011) "Fact and Fiction in EU-Governmental Economic Data". *German Economic Review* **12**(3), 243-255.

Appendix

The Brazilian Federal Court of Accounts audit of Amazon Arena 2014 World Cup soccer stadium

We present the result of TCU's audit in Table A1. Note that TCU aggregated several observation of one same item that appeared at different moments in the winning bid worksheet. We added the right-most column to make clear how to make the two-digit comparison. The original detailed budget worksheet sheet is available upon request to the authors.

The first column of Table A1 describes the items. For the sake of simplicity and space we kept the original description in Portuguese and dropped part of it when the descriptions were too long. In that case we replace the final part of a description with the symbol […]. The line in a different color corresponds to the bundled items related to air conditioning. Because different items where bundled together for analysis, we could not apply Benford's Law the bundled items.

Note that some items were not audited. These corresponded to the following two-digit categories: 14, 18, 19, 20, 30 and 31. Had our methodology been followed, the items corresponding to the to-digit categories 14, 15 and 18 would have been audited. Their combined budget was: R$4,916,251.58 (US$2,633,235.79).

Table A1 - The Brazilian Federal Court of Accounts Audit of Amazon Arena 2014 World Cup soccer stadium, February 2010

ACTIVITY (ITEM)	UNIT	Executive project budget			TCU audit			Final overprice	Two-digit category
		Qunatity	Unit price	Total price	Quantity	Unit price	Total price		
Cobertura em balanço com malha de vigas de aço intertravadas [...]	KG	3,510,000.00	22.00	77,224,674.84	3,510,000.00	22.00	77,224,674.84	0.00	77
Fachada em malhas "x" de vigas intertravadas para revestimento [...]	KG	2,749,000.00	22.00	60,481,661.29	2,842,720.00	22.00	62,539,840.00	-2,058,178.71	60
Administração local - projeto executivo	URAL	36.00	1,670,300.47	60,130,816.75	36.00	1,054,170.93	37,950,153.48	22,180,663.27	60
Membrana têxtil em fibra de vidro PTFE	M2	31,000.00	1,683.07	52,175,170.00	31,000.00	1,398.33	43,348,146.55	8,827,023.45	52
Ferragem de aço Ca-50 a	KG	5,108,143.68	8.32	42,474,214.69	4,971,054.73	7.29	36,238,988.98	6,235,225.71	10, 11, 12, 13, 16, 19, 20, 23, 24 (three different codes), 25, 34, 42, 59, 65, 67, 78 and 94.
Concreto fck 40 mpa alto desempenho (CAD) com adição de microssílica e fibra de polipropileno	M3	30,847.21	816.80	25,196,001.14	30,847.21	626.85	19,336,575.68	5,859,425.46	41, 40 (two different codes), 35, 27, 24, 23, 22, 15, 13, 12, 11 and 10
Assento retrátil - geral	UN	40,761.00	383.69	15,639,588.09	40,554.00	373.20	15,134,752.80	504,835.29	15
Projeto executivo	CJ	1.00	14,823,440.85	14,823,440.85	1.00	15,450,000.00	15,450,000.00	-626,559.15	14
Concreto especial estaca hélice - fck 20 mpa auto-adensável	M3	17,626.32	810.42	14,284,722.25	12,292.82	653.65	8,035,201.79	6,249,520.46	13 and 63

Forma plana aparente chapa compensada plastificada de 18 mm, com [...]	M2	139,441.17	91.08	12,700,301.77	139,441.17	45.27	6,312,501.77	6,387,800.00	30, 27, 22, 21, 16 and 90
Serviços agrupados do sistema de ar condicionado (excluindo os dutos)	TR	652.00	13,365.95	8,714,596.22	652.00	9,357.04	6,100,787.87	2,613,808.35	-
Concreto pré-moldado fck 40 mpa alto desempenho [...]	M3	7,378.56	1,389.28	10,250,885.84	6,925.45	1,182.39	8,188,582.83	2,062,303.01	48 and 53
Concreto fck=35 mpa	M3	10,373.80	785.82	8,161,718.53	10,373.80	594.56	6,167,846.53	1,993,872.00	14, 16 and 78
Demolição mecanizada de estrutura de concreto armado, exceto pisos [...]	M3	23,846.83	327.53	7,810,552.23	23,846.83	79.48	1,895,352.29	5,915,199.94	78
Transportes - projeto executivo	MÊS	36.00	205,763.26	7,407,477.36	36.00	190,511.39	6,858,409.99	549,067.37	74
TRANSPORTE, LANÇAMENTO E ESPALHAMENTO DE MATERIAL ESCAVADO [...]	M3	325,934.00	21.85	7,121,657.90	325,934.00	18.69	6,093,153.19	1,028,504.71	71
Cimbramento metálico	M3	180,118.07	34.02	6,127,616.74	180,118.07	38.62	6,956,159.67	-828,542.93	11 (two different codes), 15, 17 (two different codes), 42, 53 and 95
Impermeabilização com manta ASF. 3 mm, tipo iii-b, El, ou similar, aderida com asfalto [...]	M2	41,649.69	144.31	6,010,466.76	41,649.69	147.45	6,141,291.64	-130,824.88	60
Locação de grua móvel sobre trilhos com altura 50m < h < 60m, lança de 55m, [...]	EQ	28.00	211,388.39	5,918,874.92	28.00	189,567.97	5,307,903.16	610,971.76	59
Dutos convencionais, em seção retangular e em chapa de aço galvanizada, e espessuras [...]	KG	51,200.00	80.82	4,137,984.00	51,200.00	22.85	1,169,920.00	2,968,064.00	41
Locação de guindaste sobre pneus, lança treliçada [...]	EQ	20.00	201,853.99	4,037,079.80	20.00	155,029.20	3,100,584.00	936,495.80	40
Dutos convencionais, em seção retangular e em chapa de aço galvanizada, e [...]	KG	83,298.00	43.63	3,634,291.74	83,298.00	20.81	1,733,250.07	1,901,041.67	26 and 95
Switch acesso, 24 portas, 10/100/1000mb, Poe full, 2xsfp 10gb mm, lc	UN	81.00	38,603.60	3,126,891.60	NOT AUDITED				31
Fornecimento e instalação de guarda corpo metálico	KG	110,080.25	28.13	3,096,557.43	NOT AUDITED				30
Estaca escavada tipo hélice contínua, com diâmetro 80 [...]	M	18,784.00	157.54	2,959,231.36	18,784.00	149.77	2,813,279.68	145,951.68	29
Forma plana aparente chapa compensada plastificada de 18 mm, [...]	M2	31,736.15	91.08	2,890,528.54	31,736.15	45.27	1,436,695.44	1,453,833.10	19 and 48
Telão para projeção (placares)	M2	80.00	34,965.03	2,797,202.40	120.00	13,798.86	1,655,862.61	1,141,339.79	27
Corte de aço (vergalhão), inclusive remoção do local [...]	KG	1,216,188.44	2.13	2,590,481.38	1,216,188.44	0.38	460,854.48	2,129,626.90	25
Forma plana aparente chapa compensada plastificada [...]	M2	27,369.42	91.08	2,492,806.77	27,369.42	45.27	1,239,013.64	1,253,793.13	15 and 91
Dutos convencionais, em seção retangular e em chapa preta,[...]	KG	36,000.00	64.69	2,328,840.00	36,000.00	38.58	1,388,985.39	939,854.61	23
Elevador sem casa de máquina 5 paradas 26	UN	6.00	360,383.13	2,162,298.78	6.00	360,383.13	2,162,298.78	0.00	21

passageiros [...]									
Switch core, xx portas sfp 10gb mm, 48 portas rj45, 2xsup, 2xfontes	UN	2.00	1,044,451.08	2,088,902.16	NOT AUDITED				20
Cubículo blindado - COM Medição (conf. Diagrama des. MAN-[...]	CJ	1.00	1,961,122.65	1,961,122.65	NOT AUDITED				19
Desmontagem / retirada de cobertura em estrutura metálica	KG	680,733.47	2.85	1,940,090.39	680,733.47	0.32	217,834.71	1,722,255.68	19
Pintura látex acrílico em parede com duas demãos, sem massa corrida.	M2	108,347.15	17.20	1,863,570.98	108,347.15	13.22	1,432,172.62	431,398.36	18
Implantação do canteiro de obras (infraestrutura / edificações / mobiliário)	M2	2,005.00	929.26	1,863,166.30	NOT AUDITED				18
Estaca escavada tipo hélice contínua, com diâmetro 60 [...]	M	14,515.00	121.10	1,757,766.50	14,515.00	106.28	1,542,654.20	215,112.30	17
Manutenção de canteiros	MÊS	36.00	46,001.69	1,656,060.84	36.00	45,653.22	1,643,515.92	12,544.92	16
Forma plana comum compensado resinado 12 [...]	M2	28,550.00	57.67	1,646,478.50	28,550.00	54.91	1,567,680.50	78,798.00	11, 29, 34, 46, 55, 59 and 77
Divisória sanitária em painéis especiais anti vandalismo, em laminado estrutural [...]	M2	2,540.76	640.19	1,646,478.50	2,540.76	549.72	1,396,706.59	249,771.91	16
Informática / telecomunicação (equipamentos / softwares / licenças)	MÊS	4.00	391,268.92	1,565,075.68	NOT AUDITED				15
Placas de gesso acartonado - acústica ET – 09.29.00 – 01	M2	7,338.10	211.70	1,553,475.77	7,338.10	108.31	794,799.88	758,675.89	15
Escavação e carga de material de 1ª categoria	M3	408,051.00	3.70	1,509,788.70	408,051.00	2.20	899,750.32	610,038.38	15
Forro modular com alta performance acústica, 625x625mm, [...]	M2	8,117.31	184.84	1,500,403.58	8,117.31	188.34	1,528,814.17	-28,410.59	15
Barramento blindado em alumínio com conexões [...]	M	420.00	3,542.88	1,488,009.60	NOT AUDITED				14
Transporte, lançamento e espalhamento de material [...]	M3	93,124.00	13.58	1,264,623.92	93,124.00	14.27	1,328,437.47	-63,813.55	12
Piso especial tipo granilite cor referência RAL7023	M2	15,967.95	71.06	1,134,682.53	15,967.95	70.21	1,121,126.69	13,555.84	11
Emassamento de parede externa com massa acrílica com duas demãos, [...]	M2	118,168.55	8.55	1,010,341.10	118,168.55	9.46	1,117,874.48	-107,533.38	10
Vidro laminado temperado espessura de 10 mm - fornecimento e instalação	M2	1,862.14	490.61	913,584.51	1,862.14	382.47	712,215.71	201,368.80	91
Carga, transporte, descarga e espalhamento do material [...]	M3	71,487.91	12.02	859,284.68	71,487.91	7.28	520,336.32	338,948.36	12 and 84
Carga, transporte, descarga e espalhamento do material [...]	M3	26,944.13	27.05	728,838.72	26,944.13	19.54	526,503.97	202,334.75	11, 16, 21, 29 and 45
Impermeabilização com manta ASF. 3 mm, tipo iii-b, El, ou similar, [...]	M2	6,629.14	110.27	730,995.27	6,629.14	97.93	649,201.14	81,794.13	73
Locação de sistema de trilhos para grua móvel sobre trilhos, incluindo: mobilização [...]	MÊS	14.00	97,714.22	1,367,999.08	14.00	97,714.22	1,367,999.08	0.00	13

Montagem e desmontagem de grua, incluindo mão de obra, equipamentos [...]	UN	2.00	356,403.46	712,806.92	2.00	211,870.09	423,740.18	289,066.74	71
Assentos rebatível vip - com estrutura metálica, assento e encosto [...]	UN	1,400.00	1,034.98	1,448,972.00	746.00	995.21	742,426.66	706,545.34	14
Assentos rebatível vip/hospitalidade, com estrutura metálica, [...]	UN	2,252.00	623.33	1,403,739.16	2,631.00	486.29	1,279,428.99	124,310.17	14
Assentos para espectadores, rebatível com estrutura [...]	UN	50.00	999.69	49,984.50	60.00	643.43	38,605.80	11,378.70	49
Assentos rebatível vip - com estrutura metálica, assento e encosto em [...]	UN	17.00	1,446.61	24,592.37	7.00	1,391.03	9,737.21	14,855.16	24
Grama sintética	M2	3,177.13	135.49	430,469.34	3,177.13	137.68	437,427.26	-6,957.92	43
Estaca escavada tipo hélice contínua, com diâmetro 30 [...]	M	17,999.00	81.60	1,468,718.40	17,999.00	56.94	1,024,863.06	443,855.34	57 and 89
Audited sample budget								**492,594,332.98**	
% total budget								**79.97%**	
Overprice								**86,544,009.11**	
% overprice								**20.87%**	

Source: Brazilian Federal Court of Accounts (TCU) and authors’ calculations.

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