Networks, Bidding Behavior and Competition in Brazilian Treasury Auctions

Costa, Sergio G. and Bugarin, Mauricio S.

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Introduction

Empirical papers on government bond auctions typically examined price dynamics and bid profits, focusing on whether specific groups or types of auctions yield higher profits.[2] [1] [3] [5]

We combined network analysis and reduced models to assess competition and bidding behavior in BNT auctions between 2019-2023. Main findings:

- Simulated competitive bidding patterns align with NTN-B and LFT networks.
- Primary dealers bid more aggressively
- Traditional and uniform-price formats linked to lower profits
- Implications: common/interdependent values & reduced winner's curse

Introduction

Policy Experiments:

- Since 2020, auction types and formats have undergone multiple adjustments, including shifts between multiple-price, hybrid, and uniform-price mechanisms for LFT and NTN-B securities.
- The hybrid and uniform-price format was reintroduced for LFTs in April 2020, discontinued in 2022, and then resumed in 2023 with dual benchmarks before non-hybrid auctions returned later that year.
- There was also the introduction of a 3-year NTN-B benchmark using a non-hybrid multiple-price format, alongside broader changes in benchmarks, auction frequency, and a return to non-hybrid uniform-price auctions.

Network Model

The first step in our analysis is to estimate networks of bidders connected by the similarity of their bids. In this research, an edge (connection) is added to nodes (bidders) if the two bid within 0.5 basis point of each other.

The weight of an edge (connection from bidder i to j) is given by the number of similar bids. A common way to mathematically represent a network is through an adjacency matrix, defined as:

$$A_{ij} = \begin{cases} w, & \text{if node } i \text{ connects to node } j \text{ with weight } w, \\ 0, & \text{otherwise.} \end{cases}$$

We also set $A_{ij} = A_{ji}$ and $A_{ii} = 0$.

Network Model

Our approach is similar to Wachs[6], a bottom-up method for group detection in which nodes transform into groups by locally optimizing a fitness function proposed by Lancichinetti et. al[4]. The fitness function f_G for a group of nodes G is represented as follows:

$$f_G = \frac{s_{\text{in}}^G}{\left(s_{\text{in}}^G + s_{\text{out}}^G\right)^{\alpha} \times |G|^{\beta}},$$

where:

- s_{in}^G: Sum of the weights of the edges within the group.
- s_{out}^G: Sum of the weights of the edges adjacent to the group.
- $|G|$: Size of the group.
- α, β: Parameters controlling the cohesion and size of the groups, respectively.

Network Model

The fitness of adding a node n to a group G can be defined as the difference in the fitness function before and after adding n:

$$f_G^n = f_{G+\{n\}} - f_G.$$

The group detection algorithm can be performed as follows:

1. Select a node n and initialize a group containing only n.
2. Select an adjacent node of n that maximizes the fitness function. If adding this node increases the group's fitness, include it in the group.
3. Repeat until no adjacent node increases the group's fitness.

Network Model

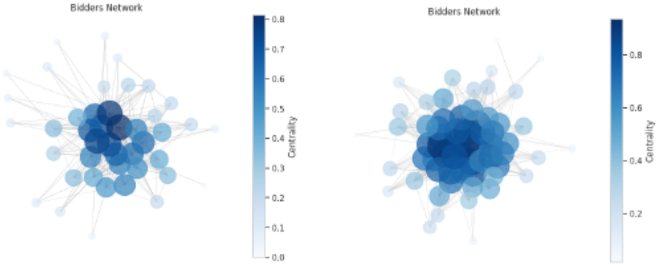
Next, we compute coherence and exclusivity of those groups.

- Coherence measures how similar connections are within a group by dividing the geometric mean by the arithmetic mean of the edge weights.
- Coherence should be near 1 if edge weights are similar within the group. If they differ, the geometric mean will be much smaller than the arithmetic, resulting in a lower coherence.
- The second metric, exclusivity, signals how much a group binds itself among its nodes in relation to outside nodes (ratio of edge weights inside the group to all group edge weights). It is close to 1 when most of the group's edge weights are connected to the own group. When most of the edge weights are connected to outside nodes, it gets closer to zero.
- A highly exclusive nodes has much more relationships within the group than with outside nodes. It is very useful for detecting clusters or communities inside networks.
- This metric is particularly useful for identifying clusters or communities within networks that have a distinct separation from other groups.
- We then simulate competitive (random networks) and collusive networks and contrast metrics with actual network data.

Data

- Auctions (2019–2023):
  - $\sim$10,000 bids for NTN-B (inflation-linked)
  - $\sim$8,000 bids for LFT (floating-rate)
- Benchmarks by maturity:
  - LFT: 3y, 6y
  - NTN-B: 3y; 5y; 7/10/15y; 20/25/30y; 40y
- Networks of bids built from bids within 0.5 bp
- Variables:
  - **Market risk:** exchange rate, VIX, CDS, Selic, etc.
  - **Benchmark-specific risk:** benchmark rate variation, amount issued, dv01, bid-to-cover, supply adjustments, number of bids
  - **Dummies:** auction type, format, bidder groups
- Accepted vs. non-accepted bids used to calibrate simulations
- Profit at the bid level is the dependent variable (for regressions)

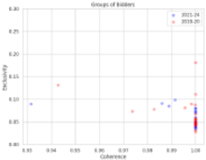
Networks Results



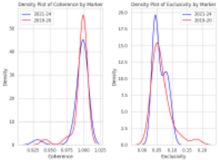
(a) NTN-B Network (b) LFT Network

Figure: Bidders Networks

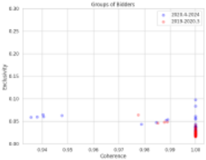
Networks Results



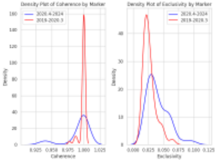
(a) NTN-B Indicators



(b) NTN-B Density

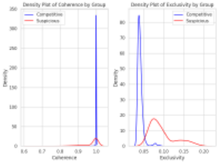


(c) LFT Indicators



(d) LFT Density

(e) Simulation Indicators



(f) Simulation Density

Networks Results

Table 36 – Test Results for NTN-B and LFT Group Metrics

Security	Test	Statistic	p-value
NTN-B	Coherence (t-statistic)	0.12	0.90282
	Exclusivity (t-statistic)	0.69	0.49222
	Coherence (U-statistic)	298.50	0.74866
	Exclusivity (U-statistic)	334.00	0.65437
LFT	Coherence (t-statistic)	2.24	0.03016
	Exclusivity (t-statistic)	-4.20	0.00008
	Coherence (U-statistic)	989.00	0.12820
	Exclusivity (U-statistic)	396.00	0.00001

Reduced-Form Model

- **Dependent variable:** $profit_{it}$ (yield change from bid to secondary market)
- **Market Risk:** macro-financial factors (exchange rate, VIX, CDS, Selic Rate)
- **Benchmark Risk:** benchmark-specific factors (rate variation, amount issued, dv01, bid-to-cover, number of bids)
- **Dummies:** auction type, formats, bidder groups, structural effects

$$profit_{it} = \beta_0 + \beta_1\, Market\_Risk_t + \beta_2\, Benchmark\_Risk_{it} + \beta_3\, Dummies_{it} + \epsilon_{it}$$

Regression Results

Table 37 – OLS Results for Multiple Specifications - NTN-B

Dependent Variable: *profit_i*	**Spec. 1**	**Spec. 2**	**Spec. 3**	**Spec. 4**	**Spec. 5**
const	0.0191*** (0.004)	0.0148*** (0.005)	0.0197*** (0.004)	0.0141*** (0.004)	0.0142*** (0.003)
anb-5	-0.0006 (0.002)	-0.0003 (0.002)	-0.0009 (0.002)	-0.0007 (0.002)	-0.0008 (0.002)
USD/BRL	-0.0043*** (0.001)	-0.0042*** (0.001)	-0.0046*** (0.001)	-0.0042*** (0.001)	-0.0042*** (0.001)
VIX	0.0003*** (0.000)	0.0003*** (0.000)	0.0004*** (0.000)	0.0004*** (0.000)	0.0003*** (0.000)
UST10Y	-0.0009* (0.001)	-0.0010* (0.001)	-0.0002 (0.000)		
SELIC	0.0002 (0.000)	0.0003 (0.000)			
dv01_of_bench	0.0000 (0.000)		0.0000 (0.000)		
number_bids	-0.0001 (0.000)	0.0000 (0.000)	-0.0001 (0.000)	0.0000 (0.000)	
ac_ratio	-0.0074*** (0.001)	-0.0073*** (0.001)	-0.0074*** (0.001)	-0.0072*** (0.001)	-0.0072*** (0.001)
bid_to_cover	0.0043*** (0.000)	0.0041*** (0.000)	0.0043*** (0.000)	0.0041*** (0.000)	0.0041*** (0.000)
d_type	-0.0158*** (0.002)	-0.0145*** (0.002)	-0.0155*** (0.002)	-0.0142*** (0.002)	-0.0143*** (0.002)
d_hybrid	0.0089*** (0.001)	0.0095*** (0.001)	0.0085*** (0.001)	0.0093*** (0.001)	0.0093*** (0.001)
d_bench3y		0.0012 (0.001)		0.0013 (0.001)	0.0013 (0.001)
d_bench20y		0.0027*** (0.001)		0.0025** (0.001)	0.0025*** (0.001)
d_bench40y		0.0018** (0.001)		0.0016* (0.001)	0.0016** (0.001)
d_dealer	-0.0002 (0.001)	-0.0003 (0.001)	-0.0002 (0.001)	-0.0003 (0.001)	-0.0003 (0.001)
d_broker_dealer	0.0017 (0.001)	0.0017 (0.001)	0.0017 (0.001)	0.0016 (0.001)	0.0016 (0.001)
d_foreign_dealer	-0.0088*** (0.001)	-0.0087*** (0.001)	-0.0088*** (0.001)	-0.0086*** (0.001)	-0.0086*** (0.001)
Observations	10510	10510	10510	10603	10603
R-squared	0.082	0.083	0.082	0.082	0.082
BIC	-47679.844	-47672.891	-47686.644	-48105.630	-48114.866

Regression Results

Table 38 – OLS Results for Multiple Specifications - LFT

Dependent Variable: *profit_i*	Spec. 1	Spec. 2	Spec. 3	Spec. 4	Spec. 5
const	0.0092*** (0.002)	0.0106*** (0.002)	0.0092*** (0.002)	0.0195*** (0.002)	0.0172*** (0.001)
anb-5	-0.0764*** (0.015)	-0.0773*** (0.015)	-0.0763*** (0.015)	-0.0774*** (0.015)	-0.0748*** (0.015)
USD/BRL	-0.0024*** (0.000)	-0.0028*** (0.000)	-0.0024*** (0.000)	-0.0038*** (0.000)	-0.0036*** (0.000)
VIX	0.0002*** (0.000)	0.0002*** (0.000)	0.0002*** (0.000)		
UST10Y	0.0010*** (0.000)	0.0011*** (0.000)	0.0009*** (0.000)		
SELIC	-0.0000 (0.000)	-0.0001 (0.000)			
dv01_of_bench	-0.0000*** (0.000)		-0.0000*** (0.000)		
number_bids	-0.0000** (0.000)	-0.0000*** (0.000)	-0.0000** (0.000)	-0.0000*** (0.000)	
ac_ratio	-0.0007* (0.000)	-0.0012*** (0.000)	-0.0007* (0.000)	-0.0018*** (0.000)	-0.0020*** (0.000)
bid_to_cover	-0.0004 (0.000)	-0.0001 (0.000)	-0.0004* (0.000)	-0.0000 (0.000)	-0.0002 (0.000)
d_type	-0.0044*** (0.001)	-0.0042*** (0.001)	-0.0044*** (0.001)	-0.0010** (0.000)	0.0001 (0.000)
d_bench3y		-0.0003 (0.000)		-0.0011*** (0.000)	-0.0010*** (0.000)
d_dealer	-0.0009*** (0.000)	-0.0009*** (0.000)	-0.0009*** (0.000)	-0.0010*** (0.000)	-0.0010*** (0.000)
d_broker_dealer	0.0010*** (0.000)	0.0011*** (0.000)	0.0010*** (0.000)	0.0011*** (0.000)	0.0011*** (0.000)
d_foreign_dealer	0.0011*** (0.000)	0.0012*** (0.000)	0.0012*** (0.000)	0.0014*** (0.000)	0.0014*** (0.000)
Observations	7361	7361	7361	7527	7527
R-squared	0.179	0.176	0.179	0.149	0.147
BIC	-53591.484	-53564.798	-53600.319	-54333.330	-54327.559

Regression Results

Table 27 – Fixed Effects and Random Effects Regression Results - LFT

Variable	Fixed Effects	Random Effects
const	0.0277*** (0.005)	0.0274*** (0.005)
d_type	-0.0004 (0.002)	-0.0001 (0.002)
ac_ratio	0.0053*** (0.001)	0.0047*** (0.001)
bid_to_cover	-0.0045*** (0.001)	-0.0044*** (0.001)
anb-5	-0.0525*** (0.019)	-0.0537*** (0.019)
USD/BRL	-0.0070*** (0.001)	-0.0069*** (0.001)
VIX	0.0002*** (0.000)	0.0001*** (0.000)
R-squared	0.323	0.325
Observations	351	351

Table 28 – Fixed Effects and Random Effects Regression Results - NTNB

Variable	Fixed Effects	Random Effects
const	-0.0185*** (0.006)	-0.0187*** (0.006)
d_type	-0.0048 (0.004)	-0.0043 (0.004)
d_hybrid	0.0119*** (0.001)	0.0116*** (0.001)
bid_to_cover	-0.0011** (0.001)	-0.0014*** (0.000)
anb-5	-0.0070** (0.003)	-0.0067** (0.003)
USD/BRL	-0.0003 (0.001)	-0.0002 (0.001)
VIX	0.0001 (0.000)	0.0001 (0.000)
R-squared	0.202	0.214
Observations	720	720

Conclusions

- Networks of bids (coherence & exclusivity) help detect patterns but require contextual interpretation
- Simulated vs. observed networks indicate competitive bidding; no evidence of collusion
- Dealers bid more aggressively (lower profits) due to obligations and privileges (e.g., greenshoe)
- Auction type and format matter: uniform-price and traditional formats yield higher revenues
- First, lower profits under uniform-price auctions suggest the presence of common or interdependent values
- Second, higher revenues under traditional auctions may reflect lower exposure to the winner's curse relative to hybrid formats
- Policy implication: competitive environment with interdependent values favors traditional and uniform-price designs

Thank you!

Sergio G. Costa — sgesteira@gmail.com

Mauricio S. Bugarin — bugarin.mauricio@gmail.com

Questions and discussion welcome

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